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FICTIONALISM AND THE PROBLEM OF UNIVERSALS IN THE PHILOSOPHY OF MATHEMATICS

ABSTRACT
Many long-standing problems pertaining to contemporary philosophy of mathematics can be traced back to different approaches in determining the nature of mathematical entities which have been dominated by the debate between realists and nominalists. Through this discussion conceptualism is represented as a middle solution. However, it seems that until the 20th century there was no third position that would not necessitate any reliance on one of the two points of view. Fictionalism, on the other hand, observes mathematical entities in a radically different way. This is reflected in the claim that the concepts being used in mathematics are nothing but a product of human fiction. This paper discusses the relationship between fictionalism and two traditional viewpoints within the discussion which attempts to successfully determine the ontological status of universals. One of the main points, demonstrated with concrete examples, is that fictionalism cannot be classified as a nominalist position (despite contrary claims of authors such as Hartry Field). Since fictionalism is observed as an independent viewpoint, it is necessary to examine its range as well as the sustainability of the implications of opinions stated by their advocates.

KEYWORDS
fictionalism, universals, realism, nominalism, philosophy of mathematics, metaphysics, ontology

Mathematics and the problem of universals

The problem of universals is one of the major ontological problems that never left the main philosophical discourse, even though the discussion revolving around it undoubtedly changed its course. It can be said that the core of this problem is one of the main preoccupations of philosophers throughout history, which, in truth, has had different manifestations. As a discipline that is essentially non-empirical, philosophy in its dealing with principles is constantly confronted with issues that are, to a lesser or greater extent, concerned with the problem of determining the status of universals. However, philosophy is not the only discipline that is not based on empirical facts. Mathematics is also a field of study that is largely made up of principles whose origins most likely cannot be found in the sensuous world.

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With the emergence of conceptualism[1] in the Middle Ages, it became clear that strictly relying on realism or nominalism is not the only way of thought. Therefore, new perspectives are emerging in this period which will not be so decisively associated to any of the two mentioned opposing viewpoints. What does this mean for mathematics? Since mathematics deals with principles, it is clear that many of its entities, like the universals, "are not to be found in the ordinary world of space and time" (Armstrong 1989: 76) and their properties cannot be talked about on the basis of immediate insight. Because of this fact, the problem of universals is very important for the philosophy of mathematics as well as mathematics alone. Determining the relationship between realism, nominalism and conceptualism has thus become one of the main tasks that philosophers of mathematics will have to solve. And they have tried, but what was immediately seen as a result was the fact that the main thing still did not change[2] – there was no consensus around which viewpoint is right. Thus, the conflicts philosophers had with the problem of determining the degree of reality of ideas are present when philosophers of mathematics are trying to determine the ontological status of many mathematical entities. This may have been anticipated, since even old Greeks have been speaking of numbers and confronted while considering their nature. Since the numbers are just "the tip of the iceberg" when mathematical entities are concerned, it was clear ever since the emergence of the philosophy of mathematics that the issue of the problem of universals would be a very fruitful topic for philosophers of mathematics.

Similarly to numbers, when it comes to mathematical concepts such as sets, points, algorithms, functions and everything else covered in mathematics, the debate on the nature of universals can be deliberately moved to its field of research without losing the original opposing positions. As a result, in the discussion of the status of mathematical entities we have realism, which in most cases is equal to platonism[3] (although it is not the only realist theory[4]), and the claim that mathematical entities exist independently of humans. On the other hand, there is nominalism[5] which claims that the abstract entities used in mathematics are essentially non-existent and that the concepts[6] derived from mathematics are the result of human aspiration to explain the empirical world. However, as Geoffrey Hellman correctly observed, the main division[7] within this problem "should not automatically be conflated with the contrast between 'platonism' and 'nominalism'" (Hellman

1 About conceptualism, as well as medieval disputes concerning the problem of universals. More in Evans (1993).

2 In relation to previous philosophical thoughts that were not necessarily concerned with determining the nature of mathematical entities.

3 See Balaguer (1998).

4 "But it must not be assumed that all realist interpretations must be platonist". (Hellman 1989: 2)

5 As the most prominent anti-realistic position. Some thinkers, such as Charles Landesman consider that nominalism is a form of so-called particularism. (Landesman 1971: 4)

6 Like all other non-empirical objects.

7 According to Landesman "same three doctrines reappear in twentieth-century surveys of the philosophy of mathematics" (realism, conceptualism and nominalism) but he claims that they are "under the new names logicism, intuitionism, and formalism" (Landesman 1971: 223-224). Although this view is interesting, we will not be dealing with it in this paper.

1989: 2). One view that is highly compatible with nominalism (if not even its theoretical embodiment) is constructivism[8] (and its most famous variation intuitionism[9]), which claims that all mathematical entities are the product of the human mind and that they have no real existence independently of it. The emergence of constructivism and other similar positions has led to the fact that the two main contradictory viewpoints (realism and nominalism) are described in the discussions in a very broad way, such that usually nominalism refutes platonism, and platonists respond to constructivists with their own arguments, it is also common for the advocates of intuitionism to respond to realists. But it seems to be most congenial for us to stick to the term realism and nominalism, which will almost entirely be synonymous with other names of related (or subordinated) positions[10].

Since this essential terminological distinction is made, there is a need to point out the connection between these two points of view. Although this may seem contradictory, realism and nominalism have one very important thing in common – their equal position on the truthfulness of the mathematical entities themselves. Both realism and nominalism treat mathematical models as something that carries a certain truth, which means that despite the existence of mutual differences in observation of the world, both positions consider that mathematical entities essentially speak of something real[11]. This is one of the rare points where there is a consensus among the followers of mathematical realism and mathematical nominalism. Does this mean that as a result of their consensus, the question of truthfulness of mathematical entities and mathematics itself is automatically treated as resolved?

Fictionalism and arguments in its favor

As far as the potential positive answer to the question posed at the end of the previous passage can be satisfactory, that is simply not the case. In the philosophy of mathematics, during the the eighties of the last century, an "autochthonous" position emerged, which would represent direct opposition to nominalism[12] and realism. Its main distinction from these two points of view is based on a different notion of the possibility of attributing truth to mathematics and its entities. The name given to it is fictionalism, the third major viewpoint[13] in the discussion regarding the nature of mathematical concepts. Its founder is Hartry H. Field who has also made a significant contribution to popularizing this position. In addition to Field, among prominent fictionalists we include David Malet Armstrong, Joseph Melia, Mark Balaguer and Stephen Yablo. All of them, despite different approaches and

8 Although some thinkers like Hellman claim that there is a "vast difference between nominalism and constructivism". (Hellman 1989: 47)

9 According to which "mathematics is an essentially subjective activity". (Øystein 2017: 76)

10 Realism for platonism, nominalism for constructivism and intuitionism.

11 Although nominalist don't believe in "realness" of mathematical entities *per se*.

12 It is important to emphasize that fictionalism is often viewed as a nominalistic position. This is not the attitude that will be favored in this paper, as it is argued that these two points are essentially different.

13 Conceptualism could be treated as a third major viewpoint, but it is not represented so much among the philosophers of mathematics.

divisions, such as those of hermeneutic and revolutionary fictionalists (Kalderon 2005: 5), had a major influence on the development of fictionalism.

Fictionalists, above all, try to present mathematics as something that does not have any reality, nor has the ability to correspond with anything from reality. According to this point of view, there are no universal ideas, forms, or any other similar concept as in realism, nor is mathematics considered to be a product of the human mind or treated as a certain construct that can be extracted from the sensuous world as in nominalism. It should, however, be taken into account that fictionalism is generally treated as a nominalist viewpoint because of its denial of reality of mathematics and its abstract entities. Armstrong states: "As a matter of fact, in the geometrical case it appears that such notions as that of a perfectly straight line or a perfectly circular object may be acquired directly in experience. For cannot something look perfectly straight or perfectly circular, even if it is not in fact so?" (Armstrong 1989: 80). Such claims about the nominalistic basis of fictionalism will be discussed below. From its very name[14] it is understood that, in contrast to, not only realism, but also nominalism, fictionalism negates any truth to mathematics[15]. Fictionalists claim that mathematics and everything it contains does not exist in any way and can be labeled as fiction. Can this controversial position be justified in any way? At first glance there is no valid reason to believe in the necessity of fictionalism, but the situation may change when we consider the arguments used by advocates of fictionalism to justify their position.

Proponents of fictionalism have tried to show the correctness of their point of view through creative examples such as the so-called paradox of existence, described by Stephen Yablo in detail (Yablo 2000: 275-312). This very simple paradox tells us that, in the same way as with the difficult problem of determining the nature of our existence, we in our theoretical limits cannot come to the knowledge of reality of abstract entities of mathematics, but our daily speech about mathematics allows us to refer to the existence of its entities. Philosophically and objectively speaking, we do not have any conclusive evidence for the real existence of mathematical entities. Our daily judgment about mathematics can be taken as something referring to existing entities. In this way, "2 + 2 = 4" testifies that abstract concepts such as numbers 2 and 4 exist, just as the statement "I exist" refers to their own existence, but only within the context of everyday speech. When it comes to philosophical discussions, we must, according to fictionalists, stick to the principles that the abstract entities of mathematics do not exist and that the mentioned speech is only a product of fiction. It seems as if fictionalists refer to this argument solely for the purpose of justifying the enormous disproportion of statements implying the existence of mathematical entities in relation to those who question their reality in everyday speech. Their pointing to the differences between philosophical and everyday discourse is certainly meaningful, but it does not say anything about our ontological commitment in everyday speech to be fictitious. "If a mathematician comes up with a radically new pure mathematical theory, she can be criticized on the grounds that the theory is inconsistent or uninteresting or useless, but she cannot be criticized — legitimately, anyway — on the grounds that the objects of the

14 Whose general acceptance is somewhat absurd.
15 And to all of its aspects.

theory do not exist" (Balaguer 1998: 56). Ultimately, one essential fact is not to be distracted from the mind, and that is there is no agreement among philosophers about the ontological status of mathematical entities.

Another interesting argument considered by Yablo is the one which refers to the thought experiment in which we imagine a certain Oracle appearing to us, telling us that mathematical entities really do not[16] exist. Since we are sure on this occasion that mathematical entities really do not exist, what would be the consequences of such knowledge? Would we suddenly stop dealing with numerous math problems and would we no longer deal with statements such as "6 is a prime number" or "triangles have three sides"? There is no doubt that, in this matter, everything would remain the same as it has been before. Our new ontological knowledge would certainly not change our way of looking at mathematics as we do now. Based on this assumption, Yablo, as well as many other fictionalists argued that our relationship with mathematics has nothing to do with its ontological status. However, it seems here that fictionalists confuse cause and effect. The fact that we can make these statements, which are meaningful and verifiable, is proof that they exist. The problem is not about finding the truth about these entities, but in the entities themselves, which, by their very existence, guarantee their truthfulness. If a certain all-knowing being tells us that they do not exist, it means that they really do not exist, but this seems thought-provoking or that it is deliberately avoiding the problem itself, because this imaginary being can be used to deny the existence of anything, even the whole outer world. This in essence does not seem much different (though the argument goes in a different way) than the famous evil spirit that René Descartes mentioned in his writings[17]. This simply cannot be a sufficient argument because it is assumed that the entities of mathematics do not exist and the consequence of this knowledge is observed, without giving any conclusive proof why the being mentioned above is correct. Although it must be acknowledged that fictionalists project a mature dose of creativity in developing such thought experiments, it all suggests that their strength is at least discernible.

Fictionalism and criticism of other positions

In addition to arguments in their favor, fictionalists also point direct criticisms to opposing viewpoints, such as realism, which, by claiming that "truth-values of our mathematical assertion depend on facts involving platonic entities that reside in a realm outside of space-time" (Field 1982: 59) acts, at first glance, as a much more stable theory than fictionalism. However, their weakest point is, ironically enough, hidden in their position about the truthfulness of mathematical claims. The problem pointed by fictionalists regarding realism is that their position has no ontological justification of its epistemic claims concerning mathematics. What does this really

16 Or *vice versa*.

17 It is about a well-known assumption that an evil spirit (Descartes 2008: 16) deceives us by affecting all our senses, such that the world around us is being questioned. However, even though we may doubt the existence of external world, we cannot doubt that we exist, since for someone's senses to deceive them must mean that someone exists. More about Descartes' analysis of skepticism in Descartes (2008).

mean? As we have already seen, realism represents theories that all mathematical entities are not only real, but also that their "realness" is something that transcends out of reality of this world because it is not limited by space or time in any sense. When considering things this way, it seems that weaknesses of this position start to become clearer, and that they can most likely be reduced to a paradoxical claim of realists that the world of ideas has nothing to do with the empirical world, but they (as empirical beings) are referring to it at the same time!

If we want to give a more detailed explanation of this problem, we could refer to the already mentioned numbers and the realist viewpoint which deals with them. Consider for example a realist who claims that number 7 exist and that we can have real knowledge about that fact. What exactly can give us proof of the existence of a number to a realist? He argues that it is a fact that he knows that this number is a part of the world of ideas in which it has immutable characteristics that guarantee its truth. It is not claimed that number 7 only exists, but it has its own clear definitions which can be talked about with immeasurable precision. The question fictionalists pose to all realists is what relies on the justification of this claim. As realism teaches us that our world is very distinct from the world of ideas in a transcendent way and that we are unable to fully understand it, how can realists then know and claim with certainty that mathematics and its elements found in the world of ideas are true? Keeping this in mind, realists seem to have "excluded themselves" on this matter, because it makes it unjustifiably possible to talk about the existence or non-existence of some entities that are an opus of some reality we cannot understand, nor will we ever be able to. It is clearly seen why the mentioned criticism that the realists have no ontological justification of their epistemic claims still stands. Knowledge is certainly conditioned by the truth, and if we cannot reach it, in spite of the contradictory claims, then we have no knowledge. Therefore, our ignorance testifies that we cannot speak of any truth because unknowable concepts (it is assumed that the world of ideas is unknowable) cannot be treated as either true or not true, rather as a product of fiction, which fictionalism claims in its core. It should be mentioned that, for fictionalists, the world of ideas is nothing more than fiction that does not have any property which would ensure its "realness". Through pointing to the unnatural relationship between claims that mathematical objects exist[18] and the fact that the world of ideas is unknowable to humans and cut out from the sensuous world, fictionalists have been able to question the basic principles of a monolithic theory such as realism.

What about the second great theory and how fictionalists observe it? Nominalistic theory, unlike realism, does not pretend to claim objective existence of mathematical entities. Nominalists see "mathematical theories as instruments for deriving nominalistically stated conclusions from nominalistically stated premises" (Malament 1982: 523). Because of this, many believe that it could be compatible with fictionalism. However, this cannot be the case because their difference is reflected in the discussion of the already mentioned essential question regarding the truthfulness of mathematics. Since nominalism claims that mathematical assumptions have truth values, it automatically differs from fictionalism. Therefore,

18 And are literal truth.

its implications can be subjected to fictionist criticism. What could be the fault with nominalism from such perspective? First of all, the objections which have a very strong foothold in realist view are not present. Contrary to the claim that mathematical entities are real, and that they exist in the world of ideas which we cannot have absolutely any knowledge about, nominalists believe that the reality of such concepts is reflected solely in our application of "their manifestations" in nature, this means that nominalism isn't ontologically committing to the assertion that abstract objects exist in the world we live in, rather we extract them from what we are given by our senses. Therefore, the same number 7, for which realists uncompromisingly claim that it must be real and independent of us and everything else that exists in the empirical world, for nominalists is not an entity that they could attribute self-existence to. Nominalists observe numbers solely through empirical application because "if a mathematical theory is added to a nominalist scientific theory, no nominalist consequences follow that wouldn't follow from the scientific theory alone" (Colyvan 2001: 69). This does not mean that nominalists claim that numbers exist fictitiously, as do fictionalists, rather their existence is strictly related to us as the creators of those abstract entities which we build with the "material" we receive from our senses. Because of this a nominalist would never say that mathematics does not possess truth values because it has this property, though nominalism does not recognize real existence to its entities.

Having in mind that nominalists do not make an ontological transgression as in realism, it would seem that funding a more serious criticism of the nominalist position would be harder than the one established in order to disprove the basic principles of realism. However, despite of all this, there is one very important aspect of nominalism that fictionalists can interpret as a big weakness of this theory. It concerns the very justification of the possibility of finding mathematical entities in the empirical world. This criticism indicates that, although the existence of some mathematical concepts may be, to a lesser or greater extent, echoed from our daily perception and that there is a very decisive possibility of pointing to them, for most such constructions we do not have confirmation from the sensuous world around us. Although we can easily describe number 7 and perceive it in various spatial extensions, things are not so simple when we start talking about abstract mathematical concepts such as derivatives, integrals, functions, etc. It is generally possible to apply the rule that the more complex mathematical theory is, the more difficult it is for the matching correspondent to be found in empiricism. In spite of this very difficult task, nominalists believe that their theory is correct because the constructions of the human mind based on mathematics and everything that makes them are so complex that, in most cases, the very mind that contructed them cannot fully understand them. This does not mean that nominalists want to mystify mathematics and its entities, nor attribute them to a real existence independent of humans, but only point to the fact that the way we perceive the world around us is sometimes so complex that we ourselves cannot interpret what it carries with itself. Therefore sometimes we encounter conceptual issues that we cannot answer, which does not mean that they do not have an empirical basis.

The very weight of such issues has indeed made a counterpoint to nominalists, but it seems that they still believe that the viewpoint they defend does not in any

way imply that talking about more complex mathematical concepts would be more problematic than talking about seemingly simpler things such as natural numbers. The unreality of numbers was also claimed by "Benacerraf, an early advocate of eliminative structuralism[19], who made much of the fact that the set-theoretic hierarchy contains many exemplifications of the natural number structure. He concluded from this that numbers are not objects" (Shapiro 2005: 22). This way nominalists defend their viewpoint, but this does not mean that fictionalists are satisfied with this response. The rebuttal of the reality of numbers and other mathematical entities does not tell us anything about the possibility of confirming the truth of mathematics. On the contrary, it distances from this idea because it mystifies human knowledge, arguing that the concepts we have created have their own independent objective confirmation in the sensuous world that surrounds us. The view that complex mathematical concepts are something unreal, but whose truth is revealed in nature and that the human mind abstracts this truth, represents a real opposition to fictionalism. Fictionalist need to find the alternative to both realist and nominalist viewpoint which are equally based on the claims regarding the truthfulness of mathematics. This problem could be posed in the following way: If one accepts the fictionalist claim that mathematics and its entities cannot be true, how do we account for evidence in our everyday life that support the fact that the truth of mathematics can be proved[20], above all, in theoretical sense?

This is a really crucial issue for positioning fictionalism in the debate on the problem of universals. It seems that the denial of the truth of mathematics and its entities is something that is less sustainable than the claim that they exist independently of us or that our minds construct them by the sensation of our senses. One cannot get rid of the impression that people from their earliest childhood discover some things that could be called mathematical truths. It is also important to note that some of these so called mathematical truths have a certain inter-subjective arrangement, which, according to many philosophers[21] guarantees conditional objectivity. Very often mathematics is used as an example of exactness[22], which intuitively acts very meaningful, given that in almost all aspects of interpersonal interaction, the truth of mathematics and its entities is not questioned. Even if we do not re-examine the essence of mathematical theories we will not argue that the claims "2 + 2 = 4" and "The square area constructed over the hypotenuse of the rectangular triangle equals the sum of the square areas constructed over the catheti of that triangle" are not true. Even mathematicians who, by complex calculations, come to the statement "2 + 2 = 5" and by means of analyzing the principle of non-euclidean geometry state that there may be certain deviations from the validity of Pythagorean theorem do not claim that the attitudes of mathematics are false or that mathematics itself is something that does not have truth values, rather

19 More about eliminative structuralism of Paul Benacerraf in Benacerraf (1965).

20 More about applied mathematics will be mentioned below.

21 Such as Immanuel Kant, who thinks that speech about "thing-in-itself", unknowable "noumena", which is the only one which could be treated as objective, is not possible. (Kant 1998: 338–353) In our empirical world the role of objectivity is taken over by intersubjectivity. More in Kant (1998).

22 Interestingly, in colloquial speech exactness is often linked with the truth.

they examine its individual principles. Fictionalists, on the other hand, believe that speech about mathematical entities cannot be true because mathematics does not exist at any level of reality.

Applied mathematics as an argument against fictionalism

When we are talking about fictionalism, we have to forget all our past intuition and what we think are indisputable facts about the nature of mathematics and its entities. Through this prism, mathematics is viewed solely as fiction, so all speech about it is treated as illusory. Is it possible that mathematics is so illusory that it has succeeded in making us believe that it is factual, and that we could not see it for so long?[23] Large number of critics would immediately recall the fact that the truth of mathematics is not only reflected in the generally accepted mathematical theories, but also in the practical application of mathematics[24]. Engineering, industry, information technology and many other areas of human activity are based on mathematics and they work very well with its principles. This is where we come to applied mathematics, which brings this discipline to a direct connection with the empirical world and deals with practical solutions to problems. "The contributions of mathematics to science (both standard and non-standard) provide solid grounds for rejecting the dispensabilist-nominalist proposals[25]" (Bangu 2012: 145). In order to solve a problem, it seems coherent that its solution must be adequate, which, of course, implies that there are also inadequate solutions, and that both are necessarily determined on the basis of concrete truth values. It is clear to everyone that the construction of the famous pyramids, as well as other, fewer or more relevant buildings, including the buildings and houses we live in, depend on the authenticity of mathematics (especially geometry[26]). If the application of these known principles of mathematics was wrongly implemented, none of these buildings could stand, and our senses prove that this is not the case. This suggests that there is a certain truth that must be attributed to mathematics and its entities, as its application has shown that the truthfulness of ultimate series of claims can be proven concisely. Even Field admits this when he said "the only serious arguments for platonism depend on the fact that mathematics is applied outside of mathematics" (Field 1989: 8). All of this acts as a very clear affirmation of the claim that by its application, namely through the successful symbiosis of applying its principles with the knowledge of the empirical world, mathematics succeeds in simultaneously removing all doubts concerning its potential of practical uselessness and gives us an immediate insight into its truthfulness. However, "Field's goal is to show that science can be done without mathematics, albeit in a terribly inconvenient manner" (Shapiro 1997: 219).

23 The very structure of this question is somewhat paradoxical, but that is what follows from the fictionalist claims about falsehood and non-existence of mathematical objects.
24 More about different approaches to practical application of mathematics in Wigner (1960) and Wilson (2000).
25 As well as fictionalist.
26 It should be noted that "nominalists often object that geometrical explanations are not genuinely mathematical". (Baker 2005: 228).

Regardless of Field's eliminative ambition[27], it seems natural to ask whether anyone can still argue that the practical application of mathematical principles "does not require its truth but only its conservativeness[28]" (Resnik 1985: 164)? In this way, not only is fictionalism re-examined, but also its connection with nominalism. If we consider the fact that the view of the authenticity of mathematics defended by nominalism differs from the fictionalist claim that it is fiction, it seems right to question their mutual relationship. Since Field's founding of fictionalism, it is considered a nominalistic position[29], but the claim itself that absolutely all mathematical aspects, including those which are directly related to sensuous world, are not true, creates the impression that fictionalism cannot be treated as a kind of nominalism. One can argue that nominalism has different variations, such as like there is extreme nominalism and its milder variations like conceptualism[30], there is one version on the opposite side of the spectrum when it comes to the view on truthfulness of mathematical entities. Although this classification is accepted in the general discourse, such criterion of determining philosophical positions can unequivocally lead to ontological relativism, in other words, the identification of fundamentally different directions of thought. If one accepts that fictionalism is a type of nominalism, it is obvious that a very important speculative maxim is ignored, and that is the one in which the approach to a specific problem that satisfies the criterion of recognizing the *differentia specifica* in relation to the object being compared to, deserves to be treated as a separate viewpoint. It does not seem very likely that many philosophers would argue that the question of determining truthfulness is not important enough for acknowledging differences to affected objects within a specific problem. Why would the matter then be different when considering the problem regarding universals, or even more precisely in the question of re-examining the essence of mathematics and its entities? It is obvious that all nominalistic viewpoints, to a lesser or greater extent, acknowledge the existence of certain truths that can be attributed to mathematics and its entities, and that fictionalism explicitly renounces them. Although for pluralism of perspectives within a single position there is a need for certain mutually opposing statements, it seems that the difference between fictionalism and other nominalist viewpoints is simply too large to allow fictionalism to be treated as a kind of nominalism.

Mathematics as useful fiction

In addition to saying that mathematics (as well as its entities) is a product of fiction, there is another important element that fictionalists attach to it, and that is its usefulness. How can mathematics be both fictional and useful at the same time? Can

27 Field wanted to "accomplish enough of an eliminativist project to avoid an ontological "commitment" to mathematical entities" (Shapiro 1997: 219).

28 "Conservativeness can in some cases be defined as "a technical property between mathematical theories and scientific theories" (Shapiro 1983: 523).

29 Field describes his point of view as nominalistic because he thinks that mathematics does not „add nothing new to the nominalistic theory" (Melia 2000: 463), although there are those who think that his nominalism can be challenged. More about Field's view and its critique in Field (2016), Malament (1982), Shapiro (1983) and Resnik (1985).

30 Assuming that conceptualism is also a kind of nominalism.

things be useful to us if they do not exist? The followers of fictionalism firmly believe that there are things that have no reality but "they could be useful fictions" (Armstrong 1989: 80). Fictionalists believe that combination of these determinations is necessary for our better understanding of mathematics and its relation to the world we live in. Take for example a fairy tale[31], which by definition is fictitious. Nobody except the followers of fictionalist realism[32] will argue that fairy tales speak of real things[33], and yet there is a general consensus that they are useful. If we take for example a fairy tale in which the main hero with his virtue and glory defeats his morally downright rivals, or if the protagonist has achieved something great due to his patience and modesty, it seems that we can still argue that there are some benefits in them. The first story tells us that it is good to be morally correct[34] and that the behaviors in accordance with moral law is something that needs to be aspired. The other one tells us that it is good to be patient and modest and that we should strive to nurture these positive traits. Both fairy tales, though indisputable products of fiction, offer us some life lessons that we can apply in our daily lives. "A metaphor has in addition to its literal content-given by the conditions under which it is true and to that extent belief-worthy — a metaphorical content given by the conditions under which it is "fictional" or pretence-worthy in the relevant game" (Yablo 2000: 249). All of this is useful, and at the same time there is no need to attribute real existence to fairy tales.

Similar to fairy tales and imagination as a creator of fiction, fictionalists believe that mathematics and its entities carry certain usefulness, but that does not make them more real than other fictional objects (Leng 2010: 155–181). Having this fictionalist argument in mind, we cannot resists the impression that we are making a big ontological leap claiming that something that does not exist can affect us by making itself useful. When it comes to fairy tales and similar fictional creations, it seems that we can find something in them that represents the analogy of the world we live in. As a result, we have people who, with their patience and modesty, have achieved their aspired goals, or the ones who have shown that moral virtue is the highest quality a human can possess in the real world[35]. On the other hand, the reality of mathematics is denied so that it cannot be analogous to anything real. As fictitious as they are, fairy tales have to be subjected to reality in a certain sense. If the fictitious abstract mathematical entities represent the subjection of reality, what is then the nature of that reality that they are inspired by? It seems as though

31 Fairy tales, like all other related literary genres, have a certain structure that completely speaks about non-existent things.

32 This view, as noted by Anthony Everett, represents the viewpoint that truthful statements can refer to fictional objects such as characters from literary works. See Everett (2005) for criticism of the fictionalist realism.

33 Although not real, it should be noted that fairy tales are very often plausible and rarely engage in contradictions. But there are also examples, such as the one from Serbian folk poetry (related to fairy tales) where Kraljević Marko "breaks the spear into three halves", which contradicts with basic mathematical principles.

34 In a coloquial sense, without deeper reflection in the deontological critique of this behavior.

35 This could be interpreted as moral realism, because it claims that the quality of a moral act can be determined on the basis of the truthfulness of the statement about it.

we have made a full circle and returned to realism, which would have to explain, with its claim about real abstract mathematical entities out of time and space, what exactly does our speech about mathematics subject to. In one hand, this is the only thing that fictionalists could call upon when they want to find what is the inspiration behind our allegedly fictitious speech about mathematics. Of course, they could, similarly to other nominalistic views, claim that mathematics is a human construct, but then they must face the problem of its truthfulness.

Conclusion

Based on the previous statements, it seems that fictionalists did not give a clear explanation that would bring us close enough to their views on the possibility of fictional mathematics that would not correspond to anything in the empirical world or any other domain of reality. Another big blow to fictionalism is the fact that there are arguments in favor of the thesis that mathematics is revealed, which is a direct attack on their assumption that mathematics is a product of human imagination. Many mathematical concepts that were thought to never be practically useful have found their application much later. Furthermore, they were actually of crucial importance for solving some of the problems of the empirical world. This is certainly something that contradicts both fictionalism and nominalism, therefore the implications of these discoveries are in favor of realism, where mathematics and its principles and entities are seen as independent of humans and the only thing we can do is to discover and apply them in the right way.

The significance of fictionalism is reflected in giving one good thought experiment to all those who deal with determining the status of mathematics and its entities. By arguing that mathematics is fiction it is brought to the same ontological level as non-existing things, so we could relate it with the fairy tales which are basically miming of the real world. All of this begs the question: What is it then that mathematics is miming? With this question we come to the knowledge that both of the alleged products of imagination have to take real entities as the basis for their structure, and thus one more question is asked: Where are these entities? As much as they attempt to attribute creative power to imagination, which it cannot possibly possess, it seems that the followers of fictionalism must acknowledge the existence of a transcendent[36] world in order for this imagination to "obtain the form" or simply accept the claims of most nominalists that, although mathematical entities do not really exist, they still tell us some truth about the empirical world we live in. Despite the fact that we have to classify fictionalism as an unjustified radical position, it was surprisingly refreshing for the discussion of the status of universals because in its essence it cannot be characterized as either realism or nominalism, even though the prevailing intellectual currents are trying to place it into the latter group of opinions. Having in mind that this is a relatively young philosophical viewpoint, it is not impossible that in the future there will be new arguments in favor of fictionalism that will try to fill the ontological gap between the correct description of the nature of mathematics and fictionalist denial of its existence.

36 Like for example Plato's world of ideas. See Ross (1951).

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Strahinja Đorđević

Fikcionalizam i problem univerzalija u filozofiji matematike

Apstrakt

Poreklo najvećeg broja problema savremene filozofije matematike se može tražiti u sporu oko određivanja prirode matematičkih entiteta kojim dominira rasprava realista i nominalista. U rubnim delovima ove diskusije se zastupaju i pojedina srednja rešenja, kao što je na primer konceptualizam. Međutim, čini se da se sve do XX veka nije pojavila treća pozicija koja ne bi iziskivala nikakvu vrstu oslanjanja na jedno od dva navedena gledišta. Tokom ovog perioda nastaje fikcionalizam, koji matematičke entitete posmatra na radikalno drugačiji način, što se ogleda u tvrdnji da su pojmovi kojom matematika barata ništa drugo do proizvoda ljudske fikcije. U ovom radu će se razmatrati odnos između fikcionalizma i dve tradicionalne pozicije u okviru diskusije koja se u svojoj srži svodi na pokušaj uspešnog određivanja ontološkog statusa univerzalija. Jedna od glavnih tačaka je i dokazivanje da se fikcionalizam ne može klasifikovati kao nominalistička pozicija (uprkos suprotnim tvrdnjama autora poput Hartrija Filda), što će biti pokazano i na konkretnim primerima. Pošto se fikcionalizam posmatra kao samostalna pozicija, a njome se spori čitav predmet matematike, nužno je preispitati njegove domete, kao i održivost implikacija stavova koje njeni zagovornici zastupaju.

Ključne reči: fikcionalizam, univerzalije, realizam, nominalizam, filozofija matematike, metafizika, ontologija