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# Mathematical self-concept: An operationalization and its empirical validity

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The major objective of this study, which used a sample of 123 ninth-grade Gymnasium (high-school) students, was to construct a psychometrically valid scale of mathematical self-concept in the Serbian language. Its main findings were that: (a) the constructed scale has good metric characteristics - high representativity and reliability and usual homogeneity; (b) the variance of mathematical self-concept is composed of two parts: one saturated with hierarchically superordinate elements of the global self-concept such as self-efficacy and intellectual-self, the other saturated with the real mathematical achievement. The relations of mathematical self-concept with some related constructs have confirmed our theoretical expectations.

*Key words:* mathematical self-concept, Math-self scale, mathematical achievement, mathematics education.

Much research has been done on relations between self-concept and school achievement. Most findings show that students with higher self-concept, i.e., students who perceive themselves as more confident in a particular area, have higher ratings of scholastic and behavioral conduct (Piers & Harris, 1964; Coopersmith, 1967; Purkey, 1970; Alban - Metcalfe & Beverli, 1981). Some authors, such as Shavelson, Hubner & Stanton (1976) and Marsh (1990), believe that self-esteem is a crucial factor in scholastic failure and success. For example, Gommage (1982) found that students' self-perceived ability was the same or better predictor of scholastic failure and success than their true ability. In general, an effort to increase positive self-evaluation might have positive effects on ability scores (Finn, 1971). For instance, Fontana & Fernandes (1994) obtained that an experimental group where a pupil self-assessment was employed, manifested significant improvements in scores on a purpose-built mathematics test when compared to a control group without this kind of assessment. Kohn (1994) claims that if our goal is to help children to become good learners or good people - or both - we should concentrate our efforts on their self-esteem.

A major component of self-esteem is self-efficacy. It was Bandura (1977, 1981) who proposed the task specificity of this component. If we accept Bandura's proposal that judgments of self-efficacy are task specific, better predictions of students' cognitive efficacy can be obtained by using measures of self-efficacy that focus on the task assessment and the underlying domain of functioning.

This study examines mathematical self-concept.

Most researchers view mathematical self-concept as a generalization of confidence in learning mathematics (e.g., McLeod, 1992). We assumed that this concept represents an organized system of beliefs supplemented by behavioural and emotional reactions regarding the value of mathematics and mathematical way of thinking as well as confidence in and motives for learning mathematics. We used this broad definition for two reasons. First, research on affect in mathematics education (e.g., McLeod & Adams, 1989; McLeod, 1992) reveals that affective variables are mutually dependent, e.g., attitudes seem to develop out of emotional responses; emotions usually occur when beliefs contradict the encountered situation; and attitudes are based upon beliefs. Second, our recent PhD study on personality in social context (Opačić, 1995) suggests that despite diversity affective variables regarding personality seem to converge to a unique pattern. We therefore hypothesized that affective variables regarding mathematics and its learning may also converge to an underlying construct which we called mathematical self-concept. Although this study is not based upon a strong theoretical framework — such framework is still missing in research on affect in mathematics education — it is our belief that it may nevertheless advance the field.

Mathematical self is included in a hierarchical model of self-concept proposed by Shavelson and his collaborators (Shavelson, Hubner & Stanton, 1976; Shavelson & Bolus, 1982; Marsh & Shavelson, 1985; Marsh, 1990). According to their model, the general self-concept is represented by a highest factor in the hierarchy. A second level in the hierarchy is occupied by academic and non-academic self-concepts. The former is built of verbal and mathematical self-concepts belonging to a lower level of the hierarchy, whereas the latter may be divided into subordinate concepts that are social, emotional and physical self-concepts. General self-concept is postulated as a stable construct. Other self-concepts are postulated as less stable constructs. This is because the more specific these concepts are, the more they depend on a particular context.

The most important findings regarding mathematical self-efficacy emerged in the last few years may be summarized as follows.

- According to Wong (1992), mathematics achievement was closely related to self-concepts and attitudes towards mathematics. As in the case of the general self-esteem, more mathematically confident students have significantly higher scores on a standardized measure of mathematics computations (Fantuzzo, Davis & Ginsburg, 1995).
- Randhawa (1993) found that the effects of mathematics attitude on mathematics achievement were mediated by self-efficacy. According to Pajares & Kranzler (1995), general mental ability and the self-efficacy had strong direct effects on performance, and general mental ability had a strong direct effect on self-efficacy.
- According to Hembree (1992), "Modest mean correlations were found between measures of performance and attitudes toward problem solving and mathematics. Confidence and self-esteem were linked at higher levels to success in problem solving." (pp. 253-4)
- Moriarty, Douglas, Punch & Hattie (1995) found that cooperative environments led not only to higher self-efficacy and achievement, but also to more appropriate behavior.
- According to Pajares & Miller (1995), students' reported confidence in solving the problems they were later asked to solve, was a more powerful predictor of that performance than was either their confidence to perform math-related tasks or to succeed in math-related courses. In addition, confidence of success in a math-related course was a stronger predictor of choice of math-related majors than was either confidence to solve mathematics problems or to perform math-related tasks.
- Skaalvik & Rankin (1995) found that both math and verbal self-perceptions were strongly related to corresponding achievement.
- According to Skaalvik & Rankin (1994), there was no differences between the sexes in mathematics achievement, but that boys had higher mathematics self-concept and self-perceived mathematics skills than girls. Furthermore, boys had higher mathematics motivation than girls, but the difference in mathematics motivation was no larger than the difference that could be explained by differences in self-perceived abilities.
- Tiedemann & Faber (1995) found gender differences in mathematics self-concept and causal attributions. Girls reported lower self-evaluations of their perceived competencies in mathematics than boys. Not only were they less likely to attribute success to their own ability than boys, but they also tended to attribute failure more to a lack of own ability and less to a lack of their own effort than boys.

The reported findings clearly suggest that mathematical self-concept is closely related to mathematical achievement, which implies that actual changes in mathematics education should address the student's self-concept as well (Davis, 1994). But, to our knowledge, there is no paper justifying the necessity of including this concept in psychological conceptual network as an independent construct. This attitude may have negative consequences for the development of psychology, since introducing new unjustified constructs may increase the number of redundant constructs. Having in mind Momirović (1997), construct justification may be based upon the following procedures:

1. Find a sufficient number of indicators (face validity).
2. Show that these indicators measure the same thing (indicator convergence).
3. Demonstrate that this construct has a position in respect to other similar or related constructs that is proposed by the underlying theory (related construct congruence - hypothetical validity).
4. Show that instruments constructed on the basis of the underlying theory discriminate subjects in accord with this theory (taxonomic discrimination/discrimination along continuum).
5. Demonstrate that the construct cannot be reduced to some other existing construct(s) (construct irreducibility).
6. Show that the developed test (the chosen collection of indicators) predicts something that is not the test itself (external validity).

Having in mind an increasing interest in studying mathematical self-concept in the last few years, its relevance to mathematics education and the lack of an appropriate scale for assessing mathematical self-concept in Yugoslav population, the major objective of this study was to construct a psychometrically valid scale of mathematical self-concept in the Serbian language. The specific research questions for this study were:

1. Can a sufficient number of indicators of mathematical self-concept be found?
2. Do the found indicators measure the same thing?
3. What is the position of the mathematical self-concept in a latent space of variables which seems to be conceptually related to it, such as the global self-concept, intellectual self, locus of control, intellectual ability and mathematical achievement?

## Method

### *Subjects*

The subjects were 123 students from four ninth-grade classes of a Gymnasium (high school). The subjects average age was 15.8 years, and 47 per cent of them were male. The subjects'mathematical abilities were mostly average, mirroring, to a large extent, mathematical abilities of ninth-grade Gymnasium students in Yugoslavia. All subjects were taught mathematics by the second author of this study, the existence of which was completely unknown to them.

### *Design*

The study primarily had a correlative design. The variables were: generalized self-efficacy, intellectual self-efficacy[1], external locus of control, non-verbal IQ (two measures were used), mathematical self, mathematical knowledge and the final mark in mathematics. The collected data were examined by scale metric feature analysis (Knežević & Momirović, 1996), correlation analysis and factor analysis.

### *Instruments*

For answering the above-mentioned questions, the following instruments were used:

1. Bezinović's (1986) scale - This instrument consisting of 10 five-grade Lykert-type items (e.g., "When I have to do something, I am usually not sure that I am able to do it"), assessed the subjects generalized self-efficacy. The instrument had been developed in Yugoslavia and used in a number of studies, with alpha reliability around .85. The alpha reliability obtained from the subjects scores was .84.
2. Opačić's (1995) scale - This instrument consisting of 13 five-grade Lykert-type items (e.g., "I easily realize relations among things and events"), assessed the subjects intellectual self-efficacy. The instrument had been developed in Yugoslavia and used in a number of studies, with alpha reliability around .85. Its alpha reliability obtained from the subjects' scores was .83.
3. A scale of Bezinović & Savčić (1989) - This instrument consisting of 10 five-grade Lykert-type items, (e.g., "In most cases, destiny determines what will happen to me"), assessed the subjects external locus of control. The instrument had been developed in Yugoslavia and used in a number of studies, with alpha reliability around .85. Its alpha reliability obtained from the subjects' scores was .81.
4. Combined solution test (Bujas, 1970) - This instrument assessed the subjects' non-verbal IQ. The instrument had been developed and standardized in Yugoslavia with high reliability. The alpha reliability obtained from the subjects' scores was .79.[2]
5. Revised Domino test D 48-b (Wolf, 1980) - This instrument assessed the subjects non-verbal IQ. The instrument had been adopted and standardized in Yugoslavia with reliability over .90. The alpha reliability obtained from the subjects scores[3] was .79.

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[1] These three variables had been chosen in accord with the reported hierarchical model of self-concept (Shavelson, Hubner & Stanton, 1976; Marsh, 1990).

[2] Intellectual abilities of Gymnasium students in Yugoslavia are mostly above average, which is, due to a low variance of the test scores, necessarily reflected on low reliability of the applied instrument.

[3] N = 93 since 19 subjects were absent from the test administration, while 11 left the classes (the school) meanwhile.

6. A Math-self scale - This instrument, which initially consisted of 59 five-grade Lykert-type items, assessed the subjects mathematical self. The final instrument, the content of which is given in Appendix I, was developed in this study. Its metric characteristics (representativity, reliability and homogeneity) are examined in the next section.

7. A ninth-grade test - This instrument assessed the subjects' mathematical knowledge on the following topics: logic and sets; combinatorics, percentage, mixtures and work; algebraic expressions; simple algebraic and geometry proofs; plane construction problems; plane isometries; linear functions, equations and inequalities and the systems of linear equations; and trigonometry of right-angled triangle. The instrument, some items of which are given in Appendix II, comprises 30 items. Its alpha reliability obtained from the subjects scores was .87.

Note that the final mark in mathematics for each subject was determinated as the arithmetic mean of his/her marks gained through the school-year. As several marks had been gained in each case, the test mark contributed to the final mark in mathematics only about 10-15 per cont. The mark could be 1 (the lowest), 2, 3, 4 or 5 (the highest).

***Procedure***

- *The administration of the applied instruments*. All instruments were administered under a group setting.

The first six instruments were administered by a group of psychologists, who found the subjects scores regarding generalized self-efficacy, intellectual self-efficacy, external locus of control, non-verbal IQ, and the math-self. Each student responded to these instruments under his/her name. The subjects were told that their answers would only be used for the development of valid personal inventories and the improvement of mathematics education. In order to maximize the validity of the given answers, the second author of this study was not involved in the administration of these instruments.

The ninth-grade test was administered by the second author and his colleagues. The author scored the test. In order to achieve a high objectivity of the assessment procedure, each student responded to each test under a password, which was replaced with his/her name after the final scores had been obtained (the results were deciphered by a group of students).

The ninth-grade test was administered at the end of a school year. Other instruments were administered on two separate occasions, three months before (instruments 1-4 and 6) and nine months after (instrument 5) this administration.

- *The construction of the Math-self scale*. This construction followed the usual procedure. Some ways and aspects of math self manifestation were defined first. Four groups of items relating to intrinsic motivation, extrinsic motivation, locus of control and subjective competency were then generated. The initial scale comprised 59 items. After its administration, content analysis was applied. Several items were removed because of their inadequate formulation and/or item redundancy. The psychometric characteristics of the remaining items were then analyzed. A number of items were removed because of their inadequate characteristics. The final scale comprised 29 items.

## Results and discussion

The representativity of the Math-self scale is reported in Table 1.

*Table 1:* The representativity of the Math-self scale

| | | |
|---|---|---|
| Kaiser, Mayer, Olkin measure of sampling adequacy | psi 1 | .94 |
| Kaiser, Rice | psi 2 | .78[4] |

In most cases we cannot include all possible indicators of some phenomena. As Table 1 shows, the chosen sample of indicators satisfactorily represent universe of all possible indicators.

Any test's score can be expressed in several different ways, such as the sum of item-scores, the first principal component score and the factor score, which represent different measurement models. Table 2 summarizes the reliability of the Math-self scale under some measurement models that are frequently used at present.

*Table 2:* The reliability of the Math-self scale

| | | |
|---|---|---|
| Reliability Under the Classical Measurement Model | | |
| Guttman | lambda 1 | .86 |
| Guttman, Cronbach $\alpha$ | lambda 3 | .89 |
| Guttman | lambda 6 | .93 |
| Reliability Measures of the First Principal Component | | |
| Lord-Kaiser-Caffrey | beta 3 | .90 |
| Measures of Reliability Under Guttman's Measurement Model | | |
| Guttman-Nicewander | rho | .94 |

[4] Kaiser-Miller's measure indicates that there might be some items with a very similar content., i.e. items characterized by very high mutual correlations and relatively low correlations with other items.

Undoubtedly, the reliability under all models satisfies the demands of psychological measurement. (As one can expect, the Math-self scale has the greatest reliability (or the smallest measurement error) under Guttman's measurement model, which treats unique variance as error variance.) Since the obtained measures of internal consistence can be used as the estimations of indicator convergence, the high reliability of the scale justifies the presumption that the chosen indicators indeed measure the same thing, i.e., that there do exist indicator convergence.

Although in the past it was considered that the measures of internal consistence are also homogeneity measures, we considered homogeneity in a different way: as a participation of the first intentional object of measurement in the total reliable variance. This participation can be expressed in different ways. Some homogeneity measures of the Math-self scale are reported in Table 3.

*Table 3:* The homogeneity of the Math-self scale

| | | |
|---|---|---|
| Mean correlation | h 1 | .22 |
| Participation of the first Guttman's factor in the total predictable (image) variance | h 2 | .50 |
| $1- (\theta^2 - \lambda^2) * (m-\lambda^2)^{-1}$ | h 5[5] | .50 |

As can be seen, the mean correlation and all other measures have the usual intensity for this type of scale. It is possible to conclude that this scale has optimal homogeneity, because a greater homogeneity might be an indicator of a low generalisibility of the measured characteristics, whereas a lower homogeneity may indicate a weak indicator convergence.

The representativity, reliability, homogeneity and internal validity of the items (indicators) are summarized in Table 4.

[5] $\lambda^2$ - the first eigenvalue of the correlation matrix; $\theta^2$ - the sum of all eigenvalues greater than 1.

*Table 4:* The representativity, reliability, homogeneity and internal validity of the items

| ITEM | REP | REL | HOM | H | B[6] |
|---|---|---|---|---|---|
| 1. I enjoy solving mathematical problems. | .99 | .74 | .79 | .82 | .78 |
| 2. When I meet an interesting mathematical problem, I cannot calm down until I have solved it. | .98 | .69 | .73 | .76 | .74 |
| 3. I am not at all interested in mathematics. | .97 | .61 | .68 | .71 | .70 |
| 4. I am always ready to solve mathematical problems. | .98 | .58 | .69 | .73 | .70 |
| 5. Solving mathematical problems can be pleasant and interesting. | .97 | .62 | .68 | .71 | .69 |
| 6. I do not usually give up solving a mathematical problem until I have found its solution. | .97 | .55 | .62 | .65 | .63 |
| 7. I am made for mathematics. | .96 | .59 | .59 | .62 | .61 |
| 8. These days, learning mathematics is a complete waste of time. | .96 | .53 | .57 | .61 | .60 |
| 9. I simply cannot do mathematics. | .96 | .50 | .45 | .48 | .49 |
| 10. Sometimes it seems I can spend all my life solving mathematical problems. | .96 | .49 | .54 | .56 | .53 |
| 11. Without a good knowledge of mathematics I will find it hard to enrol in the college I wish. | .93 | .51 | .50 | .54 | .54 |
| 12. A knowledge of mathematics gives a base for sound thinking in everyday life. | .94 | .47 | .51 | .55 | .54 |
| 13. A solid mathematical knowledge opens more possibilities when selecting a future profession. | .95 | .47 | .49 | .54 | .54 |
| 14. I am more successful than most students of my age at solving mathematical problems | .95 | .52 | .50 | .53 | .51 |
| 15. A mathematical way of thinking impoverishes human life. | .89 | .48 | .40 | .44 | .46 |
| 16. Sometimes, even after a class, I think about a mathematical problem that I could not solve in it. | .95 | .47 | .46 | .49 | .51 |
| 17. I do not try to solve a task if it appears too difficult. | .89 | .38 | .35 | .39 | .41 |
| 18. When I begin solving a mathematical problem, I suspect in advance that I will not finish it successfully. | .84 | .33 | .28 | .32 | .35 |

6 REP = $(\Sigma^{n}_{j=1} a_j^2) / (\Sigma^{n}_{j=1} r_j^2)$ where a = the column elements of matrix A= $UR^{-1}U$ $U^2 = (\mathrm{diag}(R^{-1}))^{-1}$ and R = the correllation matrix / REL - the item variance explained by other items / HOM - the proportion of the first image factor in the total image variance of the item / H - the correlation with the first principal component / B - the correlation with the total score

| ITEM | REP | REL | HOM | H | B |
|---|---|---|---|---|---|
| 19. You cannot deal with anything seriously today without a good mathematical knowledge. | .91 | .47 | .39 | .43 | .43 |
| 20. No matter how much I try, I cannot essentially influence my success in mathematics. | .82 | .41 | .28 | .31 | .35 |
| 21. I get upset when I cannot solve a mathematical problem. | .87 | .43 | .37 | .39 | .39 |
| 22. If I cannot solve a mathematical problem in 10-15 minutes, I cannot solve it at all. | .86 | .31 | .24 | .26 | .30 |
| 23. I admire people who know mathematics well. | .91 | .46 | .40 | .43 | .43 |
| 24. Success in mathematics depends on good or bad luck to a great extent. | .87 | .30 | .26 | .29 | .33 |
| 25. Good mathematicians are highly esteemed in society. | .94 | .36 | .37 | .41 | .42 |
| 26. I feel proud when I solve a harder mathematical problem. | .91 | .52 | .41 | .43 | .43 |
| 27. Success in mathematics can only be achieved by regular study and practice. | .91 | .40 | .35 | .38 | .40 |
| 28. The mark in mathematics mostly depends on the teacher's good or bad mood. | .82 | .34 | .27 | .30 | .34 |
| 29. For success in life today, it is sufficient to know four basic arithmetic operations. | .84 | .35 | .26 | .28 | .29 |
| ITEM | REP | REL | HOM | H | B |

According to Table 4, representativity, reliability, homogeneity and internal validity of the majority of the items satisfy all psychometric standards.

Undoubtedly, the Math-self scale has excellent internal metric characteristic, and may by used as a good measure of the mathematical self-concept.

What can we conclude about the construct called mathematical self-concept?

It is possible to find a lot of indicators of this construct. Although the subjects differed in the degree of their agreement with the given statements about mathematics, their answers did converge to the unique pattern. However, the indicator convergence is a necessary but not sufficient condition for the existence of some psychological construct.

What follows deals with the position of the mathematical self-concept in a latent space of the chosen variables.

The intercorrelations among the variables are presented in Table 5.

*Table 5:* Intercorrelations among the variables[7]

| VARIABLE | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 1. generalized self-efficacy | -.45 | .23 | -.18 | -.00 | -.32 | -.18 | -.21 |
| 2. intellectual self-efficacy | | -.16 | .11 | -.01 | .38 | .05 | .03 |
| 3. external locus of control | | | -.15 | -.01 | -.23 | -.27 | -.21 |
| 4. non-verbal IQ (CST) | | | | .50 | .14 | .40 | .37 |
| 5. non-verbal IQ (D48-b) | | | | | .11 | .21 | .15 |
| 6. mathematial self | | | | | | .39 | .34 |
| 7. mathematial knowledge | | | | | | | .86 |
| 8. final mark in mathematics | | | | | | | |

According to Table 5, the mathematical self was connected with related constructs (variables 1-3). It was also connected with indicators of mathematical achievement (variables 7 & 8), which is in accord with a number of studies (e.g., Wong, 1992; Fantuzzo, Davis & Ginsburg, 1995; Pajaras & Kranzler, 1995; Pajaras & Miller, 1995; Skaalvik & Rankin, 1995). Furthermore, the mathematical self was the same or even better predictor of mathematical achievement than the non-verbal IQ.

The structure of relations presented in Table 5 also suggests that the mathematical self may be primarily influenced by the global self-esteem and mathematical achievement. This hypothesis was confirmed by the results of a factor analysis dealing with variables 1-8. These results are summarized in Tables 6-8.[8]

*Table 6:* The total variance explained by the extracted components[9]

| COMPONENT | Eigenvalue | % of variance | Cumulative % |
|---|---|---|---|
| factor 1 | 2.77 | 34.7 | 34.7 |
| factor 2 | 1.55 | 19.3 | 54.0 |
| factor 3 | 1.22 | 15.2 | 69.2 |

[7] According to one-sample Kolmogorov-Smirnov test, each of variables 1 and 3-7 came from a normal distribution, whereas the distributions of variables 2 and 8 were not normal. This was expected since: (a) most subjects considered themselves as intellectually efficient persons; (b) the final mark of most subjects was 2.

[8] As variables 2 and 8 did not come from a normal distribution, each of variables 1-8 was normalized. The factor analysis of the normalized scores yielded the very same pattern.

[9] The extraction method was principal component analysis, with Guttman-Kaiser's criterion.

*Table 7:* The structure matrix[10]

| VARIABLE | factor 1 | factor 2 | factor 3 | $h^2$ |
|---|---|---|---|---|
| 1. mathematical knowledge | .94 | | | .90 |
| 2. final mark in mathematics | .92 | | | .87 |
| 3. external locus of control | -.48 | .46 | | .37 |
| 4. intellectual self-efficacy | | -.83 | | .74 |
| 5. generalized self-efficacy | | .80 | | .64 |
| 6. mathematical self | .46 | -.65 | | .51 |
| 7. non-verbal IQ (D48-b) | | | .88 | .77 |
| 8. non-verbal IQ (CST) | | | .83 | .74 |

The space of variables 1-8 had three clearly defined dimensions (Tab. 7). The first factor was a factor of mathematical achievement, the second factor was a factor of general self-concept, and the third factor was a factor of intellectual ability. Undoubtedly, it was the general self-concept (the global self-efficacy and the intellectual self-efficacy) that was a major origin of the mathematical self variability. The second part of this variability was probably caused by the reverse impact of success in mathematics on the mathematical self. These findings are in accord with Bandura's hypothesis that task-specific efficacy is a better predictor of task achievement than global self-evaluation. The question is whether mathematical self influences mathematical achievement, or mathematical achievement influences mathematical self, or both. The findings also revealed that the mathematical self, as a hierarchically lower and more specific aspect of self-evaluation, obtained a greater part of its variance from hierarchically superordinate, more global, and therefore more stable, aspects of self-evaluations.

*Table 8:* The component correlation matrix

| COMPONENT | factor 2 | factor 3 |
|---|---|---|
| factor 1 | -0.26 | .16 |
| factor 2 | | -.04 |
| factor 3 | | |

[10] The rotation method was direct oblimin with Kaiser normalization.

The correlations among the extracted factors showed that the factor of general self-concept correlated with the factor of mathematical achievement. As the factor of the general self-concept did not correlate with the factor of intellectual ability, we have grounds to require the introduction of the mathematical self-concept into predictive batteries that mostly use intelligence as a single predictor. Furthermore, we have grounds to require that actual curricular changes in mathematics education should also address the student's self-concepts (e.g., Davis, 1994). This is a relevant question since mathematics instruction may (unintentionally) contribute to a decline in mathematical self-concept (e.g., Sax, 1994), which may in turn cause an unwanted decrease in learning outcome.

Overall, the factor analysis showed that the mathematical self-concept has a theoretically presupposed position in a space of related constructs, which justifies the modality congruence of this concept.

## Concluding Remarks

This study confirmed that: (a) it is possible to find a sufficient number of congruent indicators of mathematical self-concept; (b) the chosen indicators represent an operationalization of a relevant psychological construct, which is the same or even better predictor of mathematical achievement than intelligence; and (c) the mathematical self is a variable with distinct complexity, the variability of which originates from the global self-evaluation and mathematical achievement. Our further research will be directed towards examining the developed operationalization in respect to: (a) taxonomic discrimination/discrimination along continuum, (b) construct irreducibility, and (c) external validity.

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## APPENDIX I - The content of the Math-self scale (in the Serbian language)

1. Uživam u rešavanju matematičkih problema.
2. Kada naiđem na neki zanimljiv matematički problem, ne mogu da se smirim dok ga ne rešim.
3. Matematika me uopšte ne zanima.
4. Uvek sam spreman za rešavanje matematičkih problema.
5. Rešavanje matematičkih problema može biti prijatno i zanimljivo.
6. Obično ne odustajem od rešavanja nekog matematičkog problema sve dok ne nađem njegovo rešenje.
7. Ja sam kao stvoren/na za matematiku.
8. U današnje vreme, učenje matematike predstavlja čisto gubljenje vremena.
9. Matematika mi jednostavno ne ide od ruke.
10. Ponekad mi se čini da bih celi život mogao/la da provedem rešavajući matematičke probleme.
11. Bez dobrog znanja matematike teško da ću uspeti da upišem fakultet koji želim.
12. Znanje matematike daje osnovu za ispravno razmišljanje u svakodnevnom životu.
13. Solidno znanje matematike otvara više mogućnosti pri izboru budućeg zanimanja.
14. U rešavanju matematičkih problema sam uspešniji/ja od većine svojih vršnjaka.
15. Matematički način mišljenja osiromašuje ljudski život.
16. Dešava se da i posle časa razmišljam o nekom matematičkom problemu koji na času nisam uspeo/la da rešim.
17. Neću ni probati da rešim neki zadatak, ako mi on izgleda isuviše težak.
18. Kada počnem da rešavam neki matematički problem, unapred sumnjam da ću to uspešno okončati.
19. Danas se ne možeš ničim ozbiljno baviti bez dobrog znanja matematike.
20. Ma koliko se trudio/la, ne mogu bitno da utičem na svoj uspeh iz matematike.
21. Uznemirim se kada ne mogu da rešim neki matematički problem.
22. Ako neki matematički problem ne mogu da rešim za 10 do 15 minuta, ne mogu da ga rešim uopšte.
23. Divim se ljudima koji dobro znaju matematiku.
24. Uspeh u matematici je u najvećoj meri posledica dobre ili loše sreće.
25. Dobri matematičari su veoma cenjeni u društvu.
26. Osećam se ponosnim/om kada rešim neki teži matematički problem.
27. Uspeh u matematici može da se ostvari jedino redovnim učenjem i vežbanjem.
28. Ocena iz matematike najčešće je rezultat dobrog ili lošeg raspoloženja nastavnika.
29. Za uspeh u životu danas je dovoljno znati četiri osnovne računske operacije.

## APPENDIX II - Sample Problems

1. Find (-3, 1] $\cup$ [0, 2] and (-$\infty$, 1) $\cap$ [0, 2). (Odredi ...)

2. Prove that $\sqrt{3}$ in an irrational number. (Dokaži da je $\sqrt{3}$ iracionalan broj.)

3. If $a \neq 2$ and $\pm 1$, simplify the expression $\left(\frac{1}{2-a} - a\right) : \frac{1-a^2}{a-2}$. (Ako je $a \neq 2$ i $\pm 1$, uprosti izraz ...)

4. Sketch an angle *pOq*, and transform it by reflection in point *S* on ray *p*. The obtained angle transform then by reflection in ray *q*. (Nacrtaj proizvoljni ugao *pOq*, a zatim ga preslikaj centralnom simetrijom u odnosu na tačku *S* koja pripada kraku *p*. Dobijeni ugao preslikaj dalje osnom simetrijom u odnosu na krak *q*.)

5. Prove that the diagonals of a rectangle are of the equal length. (Dokaži da su dijagonale pravougaonika jednake.)

6. Solve the inequality $x^2 < x$. (Reši nejednačinu ...)

7. Calculate $\tan x$, if $\sin x = \frac{3}{5}$. (Ako je $\sin x = \frac{3}{5}$, koliko je $\tan x$?)

## Matematičko samopoimanje: operacionalizacija i empirijska validacija skale

GORAN OPAČIĆ
ĐORĐE KADIJEVIĆ

Osnovni cilj istraživanja, realizovanog na uzorku 123 učenika prvog razreda gimnazije, bio je konstruisanje psihometrijski validne skale matematičkog selfa na srpskom jeziku. Istraživanje je pokazalo da: (a) konstruisana skala ima dobre metrijske karakteristike - visoku reprezentativnost i pouzdanost i uobičajenu homogenost, (b) varijansu matematičkog samopoimanja (mathematical self) čine dva dela: jedan zasićen hijerarhijski nadređenim elementima globalnog samopoimanja (global self) poput samoefikasnosti (self-efficacy) i intelektualnog samovrednovanja (intellectual self), a drugi zasićen realnim matematičkim postignućem. Relacije matematičkog samopoimanja sa dodirnim konstruktima potvrdile su teorijska očekivanja.

*Ključne reči:* samoprocena matematičkih sklonosti, samopoimanje, matematičko postignuće, Skala matematičkog samopoimanja, podučavanje u matematici.