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GÖDEL AND THE LANGUAGE OF MATHEMATICS[1]

Abstract: *The aim of this paper is to challenge Hao Wang's presentation of Gödel's views on the language of mathematics. Hao Wang claimed that the language of mathematics is for Gödel nothing but a sensory tool that helps humans to focus their attention on some abstract objects. According to an alternative interpretation presented here, Gödel believed that the language of mathematics has an important role in acquiring knowledge of the abstract mathematical world. One possible explanation of that role is proposed.*[2]

Keywords*:* Gödel, platonism, concepts, language, proof, understanding

1. Introduction

What is the importance of language in mathematics? Are mathematical truths independent of the language in which they are expressed? Can we acquire mathematical knowledge without using language? What is the function of language in acquiring mathematical knowledge?

Gödel has made only a few remarks about language, which makes it difficult to say with certainty how he would respond to these questions. However, Gödel's well-known interpreter, Hao Wang, does ascribe to Gödel one particular view that would answer these questions. According to this view, the language of mathematics has a purely practical role to help us, finite sensible beings, to get in touch with the world of abstract objects, which is the subject matter of mathematics.

This view does not accord well with Gödel's interest in language, which is particularly evident in his results concerning the relationship between the syntax of a formal language and its semantics, that is, concerning the completeness or incompleteness of some formal systems. This fact casts doubt on Hao Wang's interpretation of Gödel. Hao Wang finds support for his interpretation in Gödel's

2 *Acknowledgements.* I am very grateful to Professor Gabriella Crocco for making Gödel's still unpublished text, *Max Phil X*, available to me and permitting to translate two fragments from it in the text below. The publishing of *Max Phil X* is part of the research project "Kurt Gödel philosopher: from logic to cosmology", directed by Gabriella Crocco and funded by the French National Research Agency as the project ANR–09-BLAN–0313. I am also very grateful to Kosta Došen and Miloš Adžić for reading the drafts of the paper and making useful comments about it. They have contributed a lot to this paper.

platonism, the view that the world which mathematics attempts to describe exists independently of humans and their language. However, it seems that Hao Wang ignores the specificity of Gödel's view, which consists of acknowledging the existence of concepts as well as of mathematical objects such as numbers and sets. Another specificity of Gödel's view, which should be significant in matters concerning language, is the idea that knowledge of mathematical objects is attained through the understanding of concepts under which these objects fall. It seems to me that we should follow Gödel's lead in attempting to find a possible role for the language of mathematics in its contribution to our understanding of mathematical concepts. As we will see, it is plausible that at least at some point in his philosophical development, Gödel thought that the role of language in our understanding of independently existing concepts is significant.

It is clear that the questions about mathematical language will depend, to a great extent, on Gödel's view on the subject matter of mathematics – its nature and mode of existence. I will, thus, begin by outlining this view.

1.1. Gödel's platonism

In sketching out Gödel's view of mathematics and its subject matter, I will be using the picture proposed by Roger Penrose, in which the reality is divided into three spheres. One of these spheres represents the world of physical objects, another represents the mental world, and the third stands for the world of abstract entities. Many philosophers would deny the existence of some of these spheres, but Gödel would acknowledge the existence of all three. There is a certain type of connection between any two of the spheres. One part of the abstract world is mirrored in mathematical and logical laws, which hold in the world of physical beings. On the other hand, one small part of the physical world, when suitably organized, gives rise to consciousness. Finally, one small part of mental activities consists of those referring to the abstract world. These are the mental activities concerned with abstract entities and their relations (Penrose, 2011, pp. 339–343).

According to Gödel, some of these entities are the subject matter of mathematical inquiry. These are, first of all, sets and concepts. Gödel seemingly thought there is an important difference between these two types of abstract entities, since he claims that „sets are objects, but concepts are not objects" (Hao Wang, 1996, p. 235, no. 7.3.12). Concepts would be „the properties and relations of things existing independently of our definitions and constructions" (Gödel, 1944, p. 128).

The central question of this paper will be the following: What is the function of language in acquiring knowledge about mathematical reality? Is language necessary, or just makes it easier for us to reach knowledge of the abstract world? Another related question concerns the nature of language. Which sphere of reality does language belong to? Does it fall completely within one of the spheres? A negative answer to this question could be a consequence of the fact that language has different aspects, which do not necessarily belong to the same sphere of reality. Language is built out of symbols that are combined according to some specified rules, but also contain meaning. Which sphere do symbols of language belong to, and where do we place its syntax and its semantics (or the meaning of its symbols)? Which aspect of language has a role in acquiring knowledge in general, and which in acquiring particular knowledge of the abstract world?

Let us see what Gödel's answers to these questions would be according to Hao Wang.

2. Hao Wang on Gödel's view of language

The view of language that Hao Wang ascribes to Gödel can be summed up in the following sentence: "Language is nothing but a one-one correspondence between abstract objects and concrete objects (namely, the linguistic symbols)" (Hao Wang, 1996, p. 180, no. 5.5.7). The role of language in mathematics is purely practical according to this view. Everything we come to know using one particular language, we can learn using another language as well. A good choice of language can merely facilitate the acquisition of that knowledge. Namely, "symbols only help us to fix and remember abstract things" (Hao Wang, 1996, p. 180, no. 5.5.9). This fixation or identification of some abstract thing should be accomplished by associating some linguistic symbol with it. According to Hao Wang, Gödel believed that relations between abstract objects could, in this way, be replaced by relations between linguistic symbols associated with these objects, so that, by studying relations between symbols, we should be able to learn about the abstract world with less effort. Language has such an important, although purely practical role, because it is made of sensory symbols. It is easier for humans to deal with sensory objects, and to study and remember their relations and associations. In this way, language forms a kind of link between the sensory and the abstract world.

What does this suggest about the nature of language? Possibly the most important feature of language according to this view is that it is made of sensory symbols. Hence, we can say that language at least partly belongs to the physical sphere. On the other hand, if language is to help us learn things about the abstract world, its symbols should be combined in a way that reflects the relations between abstract objects. This means that rules for combining symbols of the language of mathematics into formulas and rules that relate formulas to one another should somehow be intrinsic to the abstract world itself. These rules should shape language into a sensory picture of the abstract world. Hao Wang does not try to explain how this picturing of the abstract world is even possible or how we come to know whether our language is truly an accurate picture of the abstract world. The answer might be related to a theory of meaning of mathematical symbols, but such a theory cannot be found among Hao Wang's remarks. This can make us wonder whether Hao Wang takes meaning to be one aspect of language at all or considers language to consist only of symbols and rules of syntax. In any case, he clearly does not try to explain what the meaning of the language of mathematics consists in. This could be one of the reasons why his view about the role of language in mathematics, grounded in a certain kind of connection between linguistic symbols and abstract objects, is not particularly appealing or informative. More precisely, what makes this view unappealing is the fact that Hao Wang leaves the supposition of a connection between the abstract world and symbols completely unjustified. It is not at all clear how that connection or association should be understood. Where and how does it take place? It seems the only plausible answer is that it takes place in the mental sphere, when the subject decides to reserve a particular symbol for some abstract object and to represent relations between abstract objects by relations between their symbols. But then we have to suppose that the subject is able to get in touch with the abstract objects and acquire knowledge of their relations without the use of language. Language only comes later, in helping him to remember those relations. This kind of view presupposes the possibility of epistemic access to the world of abstract entities that is unmediated by language. Some of Hao Wang's claims do point in that direction: "Since Gödel believes that we are capable of intuitions of conceptual relations, for him language plays only a minor role" (Hao Wang, 1996, p. 329). Gödel did write about mathematical intuition as the main source of mathematical knowledge, but it is questionable whether he thought that this knowledge is unmediated by language. One possible analysis of his views on the nature of mathematical intuition will be presented later in the paper.

The adequate view concerning a language should characterize its function in a correct way. According to Hao Wang's view, the basic function of the language of mathematics would be the naming of abstract objects. This function should make some others possible, such as making assertions or deducing. It seems to me that this consequence of Hao Wang's interpretation actually reveals its inadequacy. Namely, following the implications of Gödel's theory of mathematical knowledge leads us to the conclusion that if the language of mathematics is to have a role in acquiring that knowledge at all, then its basic function should not be naming, but rather making assertions and deducing. This should follow from the nature of mathematical knowledge which, according to Gödel, is conceptual. The rest of the paper will try to justify this claim. To that end, some of Gödel's results that had probably influenced him most in forming his views on the nature of mathematical knowledge and the function of language will be sketched first.

3. Gödel on ways in which mathematical knowledge is attained

3.1. The completeness and incompleteness theorems

Someone who considers the language of mathematics as merely a tool to help humans would not think that investigations of that language are important for logic or mathematics. It seems that Gödel was not of that opinion. Namely, his best-known and most appreciated results concern formal languages – more precisely, the relationship between the syntax of a formal language and its semantics. These results are his proof of completeness of the first-order predicate calculus, and his proof of incompleteness of formal arithmetic and its extensions. The question to which these results provide an answer is whether all true propositions about some subject could be expressed in one formal language, and deduced as the theorems of one formal system. According to Gödel's incompleteness theorems, when it comes to all mathematical (or even only arithmetical) truths, this cannot be done. His first incompleteness theorem states that for every consistent formal system that contains arithmetic, there will be some sentence of that formal system, which will be true, but unprovable in that system. According to Gödel's second theorem, one sentence of this kind will be the sentence that claims the consistency of the given formal system. So, if we are to prove the consistency of some formal system, we will have to use some mathematical and logical knowledge which is not formalized within that system.

What Gödel's incompleteness theorems show is that no formal system containing every true mathematical proposition can ever be made. Can we conclude anything from this about the role of language in mathematics? Gödel thought so. Due to these results, Gödel was able to give a very strong argument against the view that mathematics refers only to linguistic expressions, i.e. formulas, and is a kind of technique for their transformation. In other words, using his incompleteness theorems, Gödel has shown that mathematics cannot be reduced to syntax. In the following section, I will sketch Gödel's argument, and try to determine what it contributes to our analysis of his views on language.

3.1.1. Mathematics is not syntax of language

The idea that mathematics can be interpreted as the syntax of mathematical language stems from logical positivism. Logical positivists have tried to show that mathematical propositions do not describe any independently existing facts. According to their view, the role of language in mathematics is not to describe anything, but to express conventional rules for the use of mathematical symbols such as numerals, connectives, symbols for functions, etc. Some sentences that contain these symbols are considered to be true owing only to syntactical rules, that is, independently of any fact or meaning of the symbols. For example, any sentence of the form A=A is true, independently of the meaning of an expression that can stand in the place of 'A'. What should be proven by logical positivists is that mathematics can be reduced to sentences of this kind, i.e., to sentences that express syntactical rules, or to their logical consequences. According to Gödel, the formalist view concerning the foundations of mathematics can be taken as one possible elaboration of this idea. We can understand this view as claiming that axioms and rules of inference, to which mathematics should be reducible, are syntactical rules which stipulate that sentences of some form (represented by axioms) are true, and that any other sentence that can be derived from axioms by some rules of inference is also true (Gödel, 1951, p. 315, fn. 23).

Gödel presents many arguments against conventionalism (Gödel, 1951, pp. 315–323; 1953/9). His main argument is that in reducing mathematics to a set of conventions (no matter what form those conventions might have) some mathematical and logical knowledge has to be used. Gödel's justification of this claim is based on his second incompleteness theorem and has the following presuppositions: rules of syntax have to be finitary (i.e. they must refer to finite strings of symbols only), and if some rules are to be accepted as conventional, it has to be proven that they do not imply any factual proposition. But in order to prove that the rules of syntax do not imply any factual proposition, we will have to prove that they are consistent, since an inconsistent set of propositions implies every proposition, including factual ones. However, for proving the consistency of the rules of syntax, we will have to use mathematical and logical knowledge which is not reducible to those rules. I will briefly sketch the reasoning that may be leading Gödel to this conclusion. Every combinatorial, finitary reasoning based on some set of syntactical rules can be represented as an induction up to some ordinal (Gödel, 1953/9, p. 343, fn. 21). Gödel believed it had been proven that the limit of finitary reasoning is the induction up to ε_0. Thanks to Gentzen's consistency proof for arithmetic, which makes use of this induction, we know that it cannot be represented in formal arithmetic. But any induction up to some smaller ordinal, and, *a fortiori*, any finitary reasoning represented by that induction, can. The conclusion is that all finitary proofs based on some set of syntactical rules can be contained within one formal system. The consistency of that formal system, however, cannot be proven in it, according to Gödel's second theorem. For its consistency proof, some mathematical and logical knowledge not contained within that system will have to be used. Since that formal system contains every proof based on finitary concepts, i.e. every proof that refers to some concrete objects (such as linguistic symbols), some transfinite, abstract concepts and axioms referring to them will be needed for the proof of its consistency. In Gödel's words: "In order to prove the consistency of classical number theory (and *a fortiori* of all stronger systems) certain *abstract* concepts (and the directly evident axioms referring to them) must be used, where 'abstract' means concepts which do not refer to sense objects, of which symbols are a special kind" (Gödel, 1951, p. 318). In another place, Gödel also says that: "... since finitary mathematics is defined as the mathematics in which evidence rests on what is *intuitive*, certain *abstract* notions are required for the proof of the consistency of number theory" (Gödel, 1958, p. 241).

The use of abstract concepts in mathematics was supposed to be justified by the reduction of mathematics to syntax. As it has been shown, however, that reduction cannot be accomplished. What this tells us is that syntax cannot amount to all of mathematical knowledge. In the perspective of Gödel's platonistic view, this result shows that not all properties and relations between abstract objects can be represented by linguistic symbols and their combinations. If the language of mathematics is, as Hao Wang thought, used for mirroring relations between abstract objects, then it cannot accomplish its task very well. Still, that does not mean that Gödel thought language and its syntax do not have an important role in mathematics. This should be evident from his insistence on the importance of formalization.

3.1.2. Gödel on formalization

According to Gödel, formalization has an important role in clarifying the foundations of mathematics. In his Cambridge lecture (Cambridge, Massachusetts), Gödel takes the problem of giving the foundations for mathematics to be falling into two different parts (Gödel, 1933, p. 45). The first is the reduction of all methods of proof to a minimum number of axioms and rules of inference, while the second is the justification of these axioms. According to Gödel, the first task has been "solved in a perfectly satisfactory way, the solution consisting in the so-called 'formalization' of mathematics, which means that a perfectly precise language has been invented, by which it is possible to express any mathematical proposition by a formula" (Gödel, 1933, p. 45). Also, the methods of proof have been made completely formal. The rules of inference, by which these methods are specified, refer only to the outward structure of formulas and not to their meaning. Thanks to that, they are completely precise and univocal. This makes it possible to specify what we mean by concepts such as 'proof' or 'provable'. It is not clear whether we can speak at all about these concepts without the specification of a formal system, since it only then becomes clear which methods of proof are allowed. It is apparent Gödel thought so, as suggested by his criticism of Finsler's proof of incompleteness. The main reason why Finsler's proof of incompleteness was not acknowledged by mathematicians, Gödel claimed, is the fact that he did not specify any formal system to which his proof would refer. Finsler had tried to show that not every mathematical truth can be proven by some logically unobjectionable method. But if we do not specify some formal system, it is not clear which methods of proof are allowed, since: "... the question of what is a 'logically unobjectionable proof' is answered differently by different mathematicians" (Gödel, 2003, p. 409).

The importance of formalization for Gödel therefore lies in the fact that it provides us with a way to specify the meaning of concepts such as 'proof' and 'provable', i.e. it helps us to clearly determine which methods of proof are allowed, and to become fully aware of their consequences. This is accomplished by representing methods of proof as methods for transforming linguistic expressions, i.e. formulas. Of course, that does not necessarily mean mathematical propositions are considered to be merely linguistic expressions without meaning. The goal is to make their meaning irrelevant to deducing logical consequences from them. The disregard for the meaning of symbols proves to be very fruitful. Although formal rules of inference refer only to the structure of formulas and not to their meaning, deduction according to these rules can elucidate and expand our knowledge of some mathematical subject.

However, as Gödel has shown, the complete formalization of mathematics is unattainable. In other words, mathematics cannot be reduced to some set of mechanical principles for transformation of formulas. Some non-mechanical procedures will always be needed, and these would "include the use of abstract terms on the basis of their meaning" (Gödel, 1934, p. 370, fn. 36).

3.2. The meaning of mathematical formulas

When it comes to the meaning of mathematical or any other language, we can discern two of its aspects: the intensional and the extensional. It is difficult to determine precisely what these aspects consist in, or to draw a sharp line between them. Yet some intuition about that difference can be gathered from examples. One example can be the term 'function'. Its intensional meaning would consist in some formula or rule by which its value for an argument can be calculated. Its extensional meaning, on the other hand, would consist in a certain set of ordered pairs, the first component of which represents an argument and the second the value of the function for that argument. Since the middle of the nineteenth century, extensional meaning has been regarded as more important than intensional in mathematics. Functions have become understood as sets of ordered pairs exclusively. That opened up the possibility of studying the abstract properties of some function without knowing the rule by which its value for some argument could be found. This extensional orientation is closely related to the concurrent revolution in mathematics that consisted in focusing on understanding rather than calculating. For this reason, the formulas that make calculations possible became neglected. (cf. Devlin, 2003).

It seems that we have reasons to believe Gödel went against the dominant orientation in considering intensional meaning to be the most important in mathematics, and thought that returning to intensional considerations would be of vital significance. However, returning to intensional considerations would not mean returning to the conception of mathematics according to which mathematics deals with calculations only. Gödel thought that intensional considerations could in fact lead to a better understanding of mathematical reality. This claim should be justified and explained in view of Gödel's platonism in the remainder of this chapter. The views presented here will be based mostly on Gödel's work on extending the finitary standpoint in arithmetic and on his ideas about set theory.

3.2.1. Gödel's consistency proof for arithmetic

As we have seen earlier in the paper, the consistency of finitary arithmetic cannot be proven by using only concrete notions, such as those which refer to linguistic symbols. The consistency proof will also require taking the meaning of these symbols into account. But which aspect of their meaning should be used, according to Gödel? It is not all that difficult to answer this question since Gödel himself proposes one consistency proof (Gödel, 1958, 1972). The central concept he uses is the concept of computable function of finite type on the natural numbers. According to Gödel, the concept of function in that proof should be understood in its intensional meaning, i.e. as "an understandable and precise rule associating mathematical objects with mathematical objects" (Gödel, 1953/9, p. 341, fn. 20). So, in his consistency proof, Gödel uses intensional meaning of the concept of function, and also the concept of intensional equality between functions. In his letter to Bernays Gödel notices that "mathematicians will probably raise objections against that, because contemporary mathematics is thoroughly extensional and hence no clear notions of intensions have been developed" (Gödel, 2003, p. 283).

That Gödel had certain doubts as to the correctness of purely extensional understanding of functions is suggested by the following: "The concept 'analytic function' is a good example of the extensional version of something intensional, namely, monogeneity. It is altogether a question whether this is the correct version. What speaks against that is that, for example, <the> Γ-function is not uniquely determined, which yields a 'disappointment' just as in the propositional calculus, which 'extensionalizes' the concept 'deduction' and maybe 'truth'" (Gödel, Max Phil X, p. [87]).

We have thus far found some evidence that Gödel thought intensional meaning is important in mathematics. But how can this be justified from a platonistic point of view? The answer might lie in Gödel's distinct version of platonism, according to which, aside from mathematical objects such as numbers and sets, the concepts are also objectively existent. This view allowed Gödel to claim that our knowledge of mathematical objects is gained through understanding concepts under which those objects fall. It appears Gödel thought that language, due to its intensional meaning, could reveal some aspects of independently existing concepts to us, and in that way improve our mathematical knowledge. It seems that we can find justification for such an interpretation in Gödel's view on set theory, which he considers the fundamental mathematical discipline.

3.2.2. Gödel on set theory

The Zermelo-Fraenkel axiomatization (ZF) represents, in Gödel's opinion, a satisfactory formalization of set theory. An interpretation of that formalism leads us to a specific view on sets and the way they are formed. According to this interpretation, sets constitute a kind of cumulative hierarchy. The universe of sets is divided into levels which reflect the order of their formation. A set can be formed only if all of its elements are already given. So, on the first level, there would only be an empty set, and any set on a higher level would be formed by applying the power-set operation to a set on a lower level. On the second level, there would be a singleton with the empty set as its only element; on the third, the set with the singleton and the empty set as its elements, etc. When all finite sets are formed in this way, it is possible to form their union, which would be an infinite set. This infinite set can then function as the basis for a new hierarchy of sets, and so on.

The process of set formation could be continued indefinitely. Every given set represents the basis for the formation of some new set, etc. It is thus possible to climb indefinitely in the hierarchy of sets, without ever reaching the set of all sets. However, the Zermelo-Fraenkel theory describes only one part of that hierarchy, and is, in that sense, incomplete. The existence of some set-theoretical propositions which are undecided by its axioms represents evidence for that incompleteness. The most famous proposition of this kind is Cantor's continuum hypothesis (CH). What is claimed by this hypothesis is that the cardinal number of the continuum is the first cardinal number bigger than the cardinal number of the set of natural numbers. According to Gödel, the fact that this hypothesis is not decided by the accepted axioms should not be taken as evidence that it does not have a truth value, but as evidence that ZF axioms do not give a complete description of the universe of sets. However, that description can be complemented by adding some new axioms to those already accepted and some of these axioms might solve CH. Therefore, the undecidability of CH does not necessarily lead to the ramification of ZF theory into one in which CH is true, and the other in which it is false. In Gödel's words: "I don't think realists need expect any permanent ramifications as long as they are guided, in the choice of the axioms, by mathematical intuition and by other criteria of rationality" (Gödel, 2003, p. 372). Some of these axioms will, for example, refer to sets formed on some very high levels in set hierarchy; that is, they will claim the existence and properties of some very large sets or very large cardinal numbers. We can hope that some such axioms would solve CH and other currently undecidable problems. However, we cannot expect that any finite axiomatization will be sufficient for a complete description of the universe of sets. This follows from the principle of set formation, by which it is always possible to form some new set to which none of the accepted axioms refer. This is in perfect agreement with Gödel's general results on incompleteness which concern every formal theory, set-theory included. Every such theory could be extended. In set theory, the extensions are possible by the addition of some new axioms. So the important question is: How can we discover new axioms in set theory? What is their justification?

According to Gödel, we can discern two forms of justification for new axioms. The first way to justify some axiom of set theory is to show that it follows from the meaning of the concept of set. Gödel believed that certain new axioms, as well as all the accepted ones, can be justified in this way. As soon as we understand the meaning of 'set', it should become obvious to us that these axioms are true. Gödel considers these axioms as possessing a kind of intrinsic necessity. We could say that they explicate the meaning of the concept of set – more precisely, the iterative aspect of its meaning (cf. Gödel, 1964, p. 260). It is crucial for Gödel to emphasize that the meaning of the concept of set is not something man-made, but completely independent of humans, their knowledge, and language.

So the central question becomes: how do we come to understand the meaning of independently existing concepts, such as the concept of set, and determine which axioms follow from it? Gödel seemed to think that understanding mathematical concepts in fact consists in understanding the meaning of mathematical terms. He says therefore: "'Trying to see (i.e. understand) a concept more clearly' is the correct way of expressing the phenomenon vaguely described as 'examining what we mean by a word'" (Hao Wang, 1996, p. 233). Gödel here refers most probably to the intensional meaning of mathematical terms. It seems that the intensional meaning of the term 'set' consists for Gödel in the concept of set, while its extensional meaning consists of all sets belonging to the set hierarchy. If that is the case, then we have found the answer to why Gödel thought the intensional meaning of language is so important in mathematics. The reason is that at least some mathematical propositions are true owing to the meaning of concepts occurring in them, and not because things happen to be such and such in the world of mathematical objects. Analysis of the intensional meaning of mathematical terms would be crucial for identifying such mathematical propositions.

But Gödel also claimed there is another way to justify some axiom – not on the basis of its meaning, but its success. Some axioms can contribute to solving some previously unsolvable problems, or can make the solutions to some problems simpler or more elegant (Gödel, 1964, p. 261). As already mentioned, there are some set-theoretical propositions, such as CH, that are undecidable by ZF axioms. Gödel held that if some axiom makes such propositions decidable and produces other positive consequences, this represents good evidence that this axiom is true, i.e. it really tells us something about the universe of sets. New axioms can have consequences not only within set theory, but on formal arithmetic as well (cf. Gödel, 1933, p. 48). According to Gödel, we could even conceive of a much higher degree of verification than this: "There might exist axioms so abundant in their verifiable consequences, shedding so much light upon a whole discipline, and furnishing such powerful methods for solving given problems (and even solving them, as far as that is possible, in a constructivist way) that quite irrespective of their intrinsic necessity they would have to be assumed at least in the same sense as any well-established physical theory" (Gödel, 1947, pp. 182–183).

But even if our reason for accepting these axioms is not our insight that they follow from the meaning of the concepts occurring in them, this is still not to say that they are not true owing to that meaning, but only that we come to know this *a posteriori*. So Gödel might have actually thought that all mathematical propositions are analytic, the only difference being in the way we discover their truth. Some of his claims confirm this supposition.

3.2.3. Mathematical propositions are analytic

Gödel deems it very important that we clearly distinguish between two possible meanings of the term 'analytic proposition'. According to the first one, an analytic proposition is true if and only if it follows from definitions of the terms occurring in it. By using these definitions, we should be able to reduce such propositions to sentences of the form: A=A. According to the second, analytic propositions are true owing to the meaning of the concepts occurring in them, meanings which are, as Gödel held, not created by our definitions, and are not in any other way dependent on our language. According to Gödel, we can only say that mathematical propositions are analytic in the second sense. Even if analytic, mathematical propositions do have content. This means mathematical knowledge is not tautological knowledge. It is the knowledge about the meaning or nature of independently existing concepts. Because our understanding of these concepts can be limited, mathematical knowledge, although analytic, can be indistinct or incomplete.

Gödel's view that mathematical propositions are analytic justifies the idea that intensional meaning of mathematical terms is the most important in mathematics. Namely, the intensional meaning of these terms should reveal some aspects of independently existing concepts, and the understanding of these concepts is what mathematical knowledge consists in. Before we continue exploring this idea, one possible objection should be answered. It is very common among Gödel's interpreters to suppose that Gödel thought there is some special cognitive power that provides us with immediate knowledge of the abstract world. This is commonly supposed to be the best explanation of Gödel's stand on mathematical intuition. However, this interpretation has been criticized. It has been shown that mathematical intuition need not be conceived as some mystical faculty granting us immediate and instantaneous knowledge of the abstract world. According to the alternative interpretation, mathematical intuition is in fact the understanding of concepts (Adžić, 2014, pp. 142–153).

3.3. Mathematical intuition

What could suggest an interpretation according to which Gödel believed mathematical intuition to be some special faculty giving us an immediate insight into the abstract world is Gödel's comparison between mathematical intuition and sense perception. For example, Gödel says that: "... despite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e. in mathematical intuition, than in sense perception..." (Gödel, 1964, p. 268). This might lead us to believe that Gödel thought mathematical intuition is a kind of perception of the abstract objects and their relations. However, the role that Gödel ascribes to mathematical intuition in developing some mathematical theory can hardly be explained from that point of view. For example, Gödel thought that mathematical intuition can justify the accepted and certain new axioms of set theory. But these axioms can hardly be justified by some kind of perception of sets. Take the axiom of extensionality, according to which any two sets with the same elements are identical. What would lead us to believe that this axiom is true is rather our understanding of the concept of set, i.e. of the extensional nature of sets. Gödel himself tried to elucidate that the acceptance of set-theoretical axioms goes hand in hand with understanding the concept of set. Therefore, it seems that the interpretation that identifies the knowledge characterized as mathematical intuition with the understanding of mathematical concepts is more plausible.

It would be useful, for the purpose of understanding Gödel's view on mathematical intuition, to have in mind the difference, highlighted by Parsons (1995, p. 65), between *intuition of* some object, and *intuition that* something is true. Gödel would consider the second kind of intuition closely related to the understanding of concepts. The following quote may suggest that he was aware of the possibility for distinguishing between these two kinds of intuition: "Sets are objects but concepts are not objects. We perceive objects and understand concepts. Understanding is a different kind of perception: it is a step in the direction of reduction to the last cause" (Hao Wang, 1996, p. 235, no. 7.3.12). Gödel seems to take the second kind of intuition to be more important. We first come to understand concepts, and in that way we also gain knowledge about mathematical objects encompassed by them (cf. Adžić, 2014, p. 110, 145). This is what Gödel could mean by the following: "It should be noted that mathematical intuition need not be conceived of as a faculty giving an *immediate* knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we *form* our ideas also of those objects on the basis of something else which *is* immediately given" (Gödel, 1964, p. 268). Gödel fails to specify what exactly is immediately given, but it is plausible that he was referring to concepts. By understanding some concept, we learn what the properties of mathematical objects encompassed by it are. It remains unclear, though, how we gain any intuition *of* mathematical objects in this way. However, Gödel did not seem to think this kind of intuition, if it exists, has any important role in arriving at mathematical knowledge. Because of this, it seems that we can ignore the possibility of that kind of intuition without the danger of misrepresenting Gödel's view.

So the relevant question about Gödel's theory of mathematical knowledge is: How do we come to understand some mathematical concept? And in what way can that understanding be further improved? There are some grounds to believe that Gödel thought our understanding of some concept can be improved by establishing and developing some theory by which the meaning of that concept should be expressed. Reflection on some formalized theory, such as finitary arithmetic or set theory, can lead us to new knowledge. In the case of set theory, this is the knowledge about the truth of some new set-theoretical axioms. Gödel believes we would not be able to understand these axioms if set theory was not developed to a certain extent: "...in order only to understand the first transfinite axiom of infinity, one must first have developed set theory to a considerable extent" (Gödel, 1953/9, p. 353, fn. 43). The understanding of some concept is neither an instantaneous and immediate insight into its content, nor is it always complete and immune to error. Certain mistakes in understanding some concept could be revealed in the course of developing a theory by which the meaning of that concept should be expressed. This was the case with so-called naïve set theory, paradoxes of which brought to light the inadequacy of our understanding of the concept of set (cf. Gödel, 1951, p. 321). But these paradoxes also pointed in the direction of how we should further develop our understanding of that concept. In this way, our understanding of the concept of set was improved by our use of it. We can understand the following claim by Gödel as emphasizing precisely that point: "What Turing disregards completely is the fact that mind, in its use, is not static, but constantly developing, i.e., that we understand abstract terms more and more precisely as we go on using them, and that more and more abstract terms enter the sphere of our understanding" (Gödel, 1972a, p. 306).

Hence, it seems that we have no reason to suppose that Gödel thought there is some special faculty granting us immediate knowledge of abstract objects, such as sets, or making us able to instantly understand the whole content of some concept. If that is the case, then the question is why Gödel talked about mathematical *intuition* at all? Why did he not use the term *understanding* instead, thus preventing any false interpretations? And what does the similarity between mathematical intuition and sense perception consist in, if not the immediacy of the knowledge they provide us with?

The main basis for Gödel's comparison between mathematical intuition and sense perception could be the fact that concepts, the understanding of which Gödel characterizes as mathematical intuition, are primitive (Adžić, 2014, p. 147). This is why we cannot hope to gain knowledge of them by analyzing or defining them in terms of some more basic concepts. The understanding of primitive concepts cannot be the result of deduction, and only in that sense can we say that it is immediate. Mathematical intuition is needed if deduction is at all possible. Gödel seemed to think that by using mathematical intuition we discover not only mathematical axioms, but also the rules of inference. For example, comparing mathematical intuition with sense perception, Gödel says: "It is arbitrary to consider 'This is red' an immediate datum, but not so to consider the proposition expressing modus ponens or complete induction" (Gödel, 1953/9, p. 359). Another similarity which, according to Gödel, exists between mathematical intuition and sense perception is that both provide us with some new knowledge. In this context, Gödel speaks about mathematical intuition as the reason. He says: "The 'inexhaustibility' of mathematics makes the similarity between the reason and the senses still closer, because it shows that there exists a practically unlimited number of independent perceptions also of this 'sense'" (Gödel, 1953/9, p. 353. fn. 43).

The analogy between sense perception and mathematical intuition might be important for Gödel mainly because it should provide him with one of the arguments for platonism. The point that Gödel tries to emphasize with that comparison might be that there are equally good reasons to believe in the world of abstract objects and concepts as there are reasons to believe in the physical world. One, perhaps basic, reason for our belief in the physical world is that by supposing that there are some independently existing physical beings which have causal influence on us, we can explain the fact that sense perceptions force themselves upon us. But there is also the need, Gödel thought, for explaining the fact that some knowledge of the mathematical world also forces itself upon us as soon as we come to understand the meaning of basic mathematical concepts. This should convince us that we truly discover rather than invent truths about mathematical objects.

It seems then that the goal of Gödel's comparison between mathematical intuition and sense perception was not to convince us of some mystical faculty which gives us immediate knowledge of the abstract world. In fact, we have good reasons to believe that, according to Gödel, the way to gain knowledge of that world is by understanding the concepts. If that is the case, then we should try to answer the question about the role of language in mathematics by considering what the role of language is in our understanding of mathematical concepts.

4. Language and concepts

The foregoing discussion was intended to point in the direction in which we should look for the role of language in mathematics. I will consider it established that Gödel believed mathematical knowledge is gained through the understanding of mathematical concepts. What we should try to determine then is a way in which this understanding can be improved by the use of language. We can begin by considering the role of formal language in our understanding of concepts.

4.1. Concept and proof

As pointed out earlier in the paper, the great importance of formal systems is due to the fact that they allow us to make the methods of proof fully precise. Only when some formal system is specified it is possible to determine whether or in which way some mathematical proposition can be proven. And mathematicians do consider formal proof to be satisfactory evidence that some mathematical proposition is true. But the way in which mathematical truth is understood from the platonistic point of view, that is, as a correct description of an independently existing abstract world, makes it difficult to explain how formal proofs can guarantee the truth of mathematical propositions (cf. Tait, 1986, pp. 341–345). To explain this from the platonistic point of view, some account of the relationship between formal systems and the abstract world would have to be given. I believe Gödel would be able to do that. What he could claim is that deduction in some formal system represents one aspect of the understanding of mathematical concepts which belong to the abstract world. First of all, we cannot build a formal system without axioms or rules of inference. It seems that Gödel would claim that we discover these axioms and rules of inference by understanding the meaning of basic mathematical concepts. We could thus say, for example, that the axioms of formal arithmetic express the meaning of the concept 'natural number'. Some aspects of that meaning are expressed by the axioms describing fundamental properties of the successor operation and others by the axioms that describe binary operations, i.e., addition and multiplication. On the other hand, set-theoretical axioms express the meaning of the concept of set, and the discovery of new axioms would represent the improvement of our understanding of that concept. Once we have arrived at certain axioms, by using the rules of inference, we can derive their logical consequences. In that way, our understanding of the concepts contained in these axioms can be improved. If axioms are true owing to the meaning of the concepts contained in them, then so are their logical consequences. These logical consequences can therefore reveal to us some new aspects of the meaning of concepts expressed by the axioms. In this way, deduction can deepen our understanding of mathematical concepts.

At one point, Gödel says that "the concept of concept and the concept of absolute proof may be mutually definable" (Hao Wang, 1996, p. 188, no. 6.1.13.). The preceding discussion can help us in grasping the meaning of this enigmatic claim. Gödel takes the concept of absolute proof to be closely related to the concept of absolute provability. Absolute provability would presuppose the provability of every true proposition of some kind. The relationship between the concept of concept and the concept of absolute provability can then be explained by the fact that a complete understanding of some concept should enable us to prove every true proposition containing it. On the other hand, if we are able to prove every true proposition containing some concept, that means we have gained a thorough understanding of that concept. However, we have seen that there is no consistent formal system of considerable strength (i.e. which contains at least formal arithmetic) within which all true propositions of some mathematical discipline can be proven. This fact raises the question whether, by speaking of absolute provability, Gödel presupposes the possibility of some methods of proof which would not be related to any formal system, but which would be possible if we were able to gain the complete understanding of some concept. It is important to answer this question, since it is related to the question of the possibility of some language-unmediated understanding of concepts.

4.1.1. Gödel on absolute provability

The concept of provability is an ambiguous concept, since which propositions are provable depends on the chosen formal system. Another epistemological concept – the concept of computability used to be in a similar way dependent on the choice of formalism. But, according to Gödel, by introducing Turing-computability we have acquired "an absolute definition of an interesting epistemological notion, i.e., one not depending on the formalism chosen" (Gödel, 1946, p. 150). Some function of natural numbers is Turing-computable if and only if it is computable in arithmetic. So thanks to the notion of Turing-computability, we can speak of the functions that are computable independently of the chosen formal system.

On the other hand, the concept of provability can never be specified in this way. As we have seen when considering formalization of set theory, there will always be some proposition, which is not provable in the given formal system, but which becomes provable if we extend that formal system by adding some new axioms to it. That is why there can be no formal system which contains every possible proof. Still, that does not mean, according to Gödel, that we cannot specify the concept of absolute provability in some non-constructive way. We just need to find a way to correlate and describe all possible extensions of set-theoretical formal systems. And this can be done. Namely, all extensions of formalism in set theory can be made by adding some new axioms to it. Although we cannot foresee what form all these axioms might have, we can characterize them in some way. "It is certainly impossible to give a combinatorial and decidable characterization of what an axiom of infinity is; but there might exist, e.g., a characterization of the following sort: An axiom of infinity is a proposition which has a certain (decidable) formal structure and which in addition is true" (Gödel, 1946, p. 151). The 'absolute provability' can then be taken to stand for derivability from these axioms. In that way we would attain a concept of provability that does not depend on our choice of formalism. But that concept would not be independent of formalisms in general. We are led in its characterization by the existing formal system and the ideas of its extensions that are suggested by this formal system itself. This concept is absolute in the sense that it embraces all the propositions expressed in the language of set theory (that is, propositions formulated by the terms 'set', '∈', and logical constants), which are provable in some formal system or its extensions. "The question whether the two epistemological concepts considered [besides provability, Gödel considers here the concept of definability], or any others, can be treated in a completely absolute way is of an entirely different nature" (Gödel, 1946, p. 153). What Gödel means by this could be the possibility of treating the concept of provability in set theory independently of the way in which set theory is formalized. It is not clear whether we can make sense at all of such a concept.

We can conclude that by absolute provability in set theory Gödel meant the provability which would be possible if we were able to gain the complete understanding of the concept of set. And that understanding would be expressed in the formal systems, by stronger and stronger axioms of infinity. This could explain the meaning of the following claim of Gödel: "The idea of proof may be non-constructively equivalent to the concept of set: axioms of infinity and absolute proofs are more or less the same thing" (Hao Wang, 1996, pp. 268–269, no. 8.4.21).

4.2. Concepts and the syntax of language

So it seems that the formal proofs do contribute to our understanding of mathematical concepts. But the only aspect of language relevant to these proofs is its syntax. The meaning of symbols of some formal language, i.e. their interpretation, is completely irrelevant for formal deductions. It follows that it is the syntax of language that can improve our understanding of the abstract concepts. The question is how is that possible. What syntax has to do with the meaning of objectively existing concepts?

There is straightforward evidence that Gödel thought that the syntax of language has an important role in improving our understanding of concepts. Namely, Gödel claimed that: "The fact that the understanding of concepts becomes significantly clearer by the construction of their sensory images [i.e., words] seems absurd at first [could the perception of some landscape become clearer by sketching a picture of it?]. But the reason might be that the material (i.e., the finite combinatorics) already contains, in some way, the image of the conceptual so that only this can really be depicted (or depicted simply). This would mean: the truth is what has the simplest and the most beautiful symbolic expression. (This means: the finite combinatorics already contains a 'picture of God')" (Gödel, Max Phil X, p. [18]).

Gödel seemingly thought that if we are to explain the fact that the syntax of language contributes to our understanding of mathematical concepts, we have to suppose that these concepts direct somehow our use of mathematical symbols. In other words, we have to suppose that the syntax of the language of mathematics is determined by its intensional meaning, i.e. by the concepts which are expressed in that language. We could understand these concepts as a kind of techniques for using the linguistic symbols. If the concepts are understood in this way, it becomes clear how language can improve our understanding of them. This idea could also help us understand the importance of the criteria usually used in the choice of a formal system, such as its simplicity or elegance. The fact that some formal system satisfies these criteria can be the sign that it really depicts some aspects of the conceptual world. That the world of concepts is independently existent is shown by the fact that there is absolutely no freedom in deriving theorems from the axioms that express the meaning of these concepts. Mathematicians cannot decide what will be the theorems of their formal system. The outcome of some deduction can come as a surprise to them. This should make us believe that by deducing in some formal system they truly discover, rather than invent, the properties of mathematical concepts which should somehow underlie that formal system (cf. Gödel, 1951, p. 314).

To sum up, Gödel would be able to acknowledge and partly explain the role of language in mathematics on account of his view that mathematical knowledge is conceptual. However, he did claim that the role of language in mathematics, and also in philosophy, should not be overestimated. It will be worthwhile to consider what could be his reasons for claiming that – it can help us to understand better his views on language.

4.3. Understanding the role of language in the right way

"The overestimation of language is deplorable" (Hao Wang, 1996, p. 180, no. 5.5.7). The context in which Gödel makes this claim might be of primary importance for understanding its intent. According to Hao Wang, it was one of the points in Gödel's criticism of Wittgenstein and his influence on subsequent philosophers. One way in which Wittgenstein influenced these philosophers is by the idea that great philosophical questions can be solved by linguistic analysis. As we have seen, it seems that Gödel thought as well that language can help us improve the understanding of concepts thanks to which we could solve some mathematical and also some philosophical problems. How then the cited claim fits with this view?

The main reason for Gödel's discontent with the use of linguistic analysis in philosophy may be the fact that the way the philosophers of his time understood the role of language in solving philosophical problems is very different from the way that role was understood by Gödel. It was common among philosophers of that time to consider language to be purely conventional. The syntax and the meaning of language were considered to be something man-made, which does not conform to any external principle or rule. Also, philosophical questions and problems were taken to be the consequence of an incorrect use of language. This is why it is believed that, by making evident the mistakes that are present in the philosophical use of language, linguistic analysis can help us resolve these questions. It seems to me that Gödel opposed exactly this conventional view on language and the proposed explanation of its role in philosophy. According to Gödel, the meaning of language is certainly not man-made, but consists in independently existing concepts, their properties and relations. Owing to this, language can have an important role in solving those philosophical or mathematical problems which we can characterize as conceptual. The reason why Gödel insisted that language is just a correspondence between symbols and abstract entities might be to emphasize that what is important about language are not its conventional aspects, but the fact that it expresses the meaning of abstract concepts. Only thanks to that language can have an important role in philosophy.

Gödel takes his main task in philosophy to be the analysis of the highest concepts (cf. Floyd, Kanamori, 2015). What should contribute to this analysis is, Gödel believed, the reading of some philosophical texts. However, "a substitute for the reading of philosophers is reading some good books with precise analyses ... learning of language [*Hebrew*, *Chinese*, Greek] and the precise *definition* of words and concepts that occur" (Gödel, Max Phil IX, p. [79], as cited in: Floyd, Kanamori, 2015). One result of the analysis of concepts should be the formal theory of concepts, which would, in Gödel's opinion, form the central part of logic. This theory should improve our understanding of the formal characteristics of concepts and help us solve so-called intensional paradoxes. Gödel does not say much about how he thinks this theory should look like and what would be the way to establish such a theory, but we can form some ideas about that following his remarks. What we should consider is what role linguistic analysis has in the theory of concepts that Gödel envisages.

4.4. The theory of concepts

Although we have some intuitions about formal concepts, such as 'concept', 'proof', 'proposition', etc., these intuitions are rather rudimentary. This should, in Gödel's opinion, be evident from the existence of so-called intensional paradoxes. Intensional paradoxes are, according to Gödel, those related to concepts and independent of the language in which these concepts are expressed. On that basis Gödel makes a difference between intensional and semantic paradoxes, which are the paradoxes of some particular language and which, Gödel believed, had been solved in a satisfactory way. An example of a semantic paradox is the one concerning the predicate 'true in this language'. This paradox arises if we ask the question whether the sentence "This sentence is not true" is true or not. However, as soon as we realize that 'true in language L' cannot be a predicate of L, the paradox is resolved. Unlike semantic paradoxes, which are related to some particular language, "conceptual paradoxes can be formulated without reference to language at all" (Hao Wang, 1996, p. 271, no. 8.5.10). An example of intensional paradox is the paradox of the concept of concepts not meaningfully applicable to themselves. This paradox arises if we ask whether this concept is meaningfully applicable to itself or not. The existence of intensional paradoxes should not, Gödel thought, raise any doubts as to the objective existence of concepts. On the contrary, it should make us believe that subjectivism is wrong – the fact that we cannot use the concepts in an arbitrary way reveals that they are not our creation. By revealing the formal characteristics of objectively existing concepts, the theory of concepts should help us resolve these paradoxes.

What would such a theory look like? Gödel suggests that in establishing the theory of concepts we should take set theory for our guide and try to identify the intensional counterparts of its axioms. The primitive relation of something being an element of some set would be replaced by the relation of something falling under some concept or the applicability of concept to something. Some important differences between these two theories would, however, exist. One difference is that, contrary to sets which cannot be the elements of themselves, some concepts, such as the concept of concept, are applicable to themselves, so the theory of concepts should allow that. The concept of concept is also the one which encompasses all the others. So, the theory of concepts should also allow for the existence of the universal concept, although in set theory there is no such thing as a universal set. However, some constraints on the applicability of concepts would have to be specified, so that the intensional paradoxes could be solved. "It might even turn out that it is possible to assume every concept to be significant everywhere except for certain 'singular points' or 'limiting points', so that the paradoxes would appear as something analogous to dividing by zero" (Gödel, 1944, p. 138). The axioms of the theory of concepts should state the properties of primitive concepts and by using these axioms we should be able to prove the properties of the concepts composed out of primitive concepts. Gödel believed that we have some proper ideas as to which concepts are primitive. As examples Gödel gives the following list of concepts: negation, conjunction, existence, universality, object, concept, the relation of something falling under some concept and so on (Hao Wang, 1996, no. 8.6.17., 9.1.26). What we still do not have is a clear intuition about the properties of these concepts which should be stated by the axioms. If we are able to cultivate our intuition of primitive concepts and to formulate axioms about them, we will come to the theory of concepts. What would be the result of such a theory? In which way could that theory improve our mathematical knowledge? It might be the case that Gödel thought the theory of concepts would elucidate the basic principles of intensional considerations which are, in his view, very much present in mathematics. So we might hope that the theory of concepts will have the role similar to that which Gödel ascribes to predicate logic. Namely, predicate logic is useful in mathematics mostly because it specifies the allowed rules of inference. But these rules are based solely on the extensional meanings of mathematical formulas. Gödel perhaps thought that the theory of concepts should provide us with some new, intensional, rules of inference.

So the basic question is: How can we cultivate our intuition of primitive concepts and discover the axioms of the theory of concepts? Did Gödel think that the intensional analysis of language can help us do that? There are actually some reasons to doubt that this is what Gödel had in mind. In what follows I will identify these reasons and try to determine whether they undermine the interpretation of Gödel's views on language presented here.

4.4.1. The role of linguistic analysis in the theory of concepts

The interpretation of Gödel's views on language presented in this paper suggests that Gödel would take language to provide guidance in establishing the theory of concepts. However, it is not so clear how could linguistic analysis lead to the theory of concepts as Gödel envisages it. First of all, it seems that Gödel was not using any linguistic criteria for grouping certain concepts as primitive. The concepts that are primitive in his view are expressed in language by different grammatical categories. Among them there are connectives (negation, conjunction, existence, universality), unary predicates (object, concept) and binary predicates (relation of something falling under some concept). It is not clear what Gödel's criterion for choosing exactly these concepts as primitive is. Worse than that, it is not even clear according to which criteria are these concepts at all. Gödel characterizes concepts as "the properties and relations of things existing independently of our definitions and constructions" (Gödel, 1944, p. 128). However, we could hardly say that negation or universality is the property of some thing or the relation between things.

If he had used linguistic criteria, he would most probably have chosen predicates as the main subject of the analysis, since properties and relations are expressed by them in language. The theory of concepts he envisaged would then deal with the intensional analysis of predicates. However, it is unlikely that Gödel had that in mind. The main motive for establishing the theory of concepts for Gödel seems to be the existence of intensional paradoxes, one of which is the already mentioned paradox of the concept of concepts not meaningfully applicable to themselves. And in fact, there is a paradox analogous to this one, but referring to predicates, known as the Grelling-Nelson paradox. Among predicates of some language there are those which do not describe themselves. Such predicates are called heterological and the examples are 'Serbian', 'unwritten', etc. The paradox arises if we ask whether 'heterological' is a heterological predicate or not. However, it is questionable whether Gödel would identify his paradox of the concept of concepts not meaningfully applicable to themselves with the Grelling-Nelson paradox. A negative answer is more probable, since Gödel insisted on the difference between intensional and semantic paradoxes, which should consist in the fact that intensional paradoxes are language independent.

There are thus some reasons to doubt that Gödel considered linguistic analysis to be the right method for establishing the theory of concepts. Another reason is Gödel's interest in Husserl's phenomenology which he presents as the method that could help in clarifying the meaning of concepts (Gödel, 1961, pp. 383–385). For some of Gödel's commentators, specifically for Parsons, Gödel's interest in the philosophy of Gotthard Günther is also symptomatic. Gödel held a long correspondence with Günther, which was mostly about Günther's neo-idealistic program of elucidating the contents of consciousness. Parsons emphasized the lack of Gödel's response to Günther's ideas in which the importance of linguistic analysis for an account of thought and its contents (such as concepts) should be stressed (Gödel, 2003, p. 475). According to Parsons and other commentators, Gödel's interest in Günther's and Husserl's philosophy was motivated by his search for the right method for establishing the theory of concepts. However, it seems that these philosophies do not ascribe much importance to language or linguistic analysis.

What this might suggest is that Gödel thought the role of linguistic analysis in the understanding of concepts is limited. The reason might be his view on the nature of concepts. Gödel insisted that the properties and relations of concepts are independent of the language in which they are expressed. Perhaps because of that Gödel thought that language cannot help us to discover all of the properties of concepts, and some of these properties could even be obscured by language. If that is the case, then, even though Gödel did ascribe some important role to language in acquiring the knowledge of concepts, he might also think that this knowledge should be supplemented by the knowledge we should try to gain using some non-linguistic methods. However, by dismissing linguistic analysis as inadequate for some kind of deeper understanding of concepts, Gödel would deprive himself from the accuracy and exactness, which linguistic analysis makes possible. As a consequence, his search for the deeper understanding of concepts could leave us with no clear expectations about the success or the importance of that undertaking.

5. Conclusion

This paper has tried to show that Hao Wang's interpretation, according to which Gödel thought language is just a sensory tool which helps humans to get in touch with the world of abstract entities, is wrong. On the contrary, it seems likely that Gödel would take language to have a substantial role in acquiring mathematical knowledge, owing to its intensional meaning, which consists in objectively existing concepts. The reason is that the understanding of precisely these concepts is what Gödel considered mathematical knowledge to consist in. However, concepts are, in Gödel's view, independent of the language in which they are expressed. Perhaps for that reason Gödel tried to warn us against the overestimation of language. This can also be the reason why Gödel might think that there are some other, non-linguistic methods by which the properties of the conceptual world could also be discovered. However, abandoning linguistic analysis in search for the deeper understanding of concepts could have some negative consequences. The fact is that we can form some intuition about what concepts might be, as long as we treat them as something which is expressed by language, or which directs somehow our use of linguistic symbols. On the other hand, if we try to consider them independently of language, we lose any intuition about them. So it is not clear anymore what some deeper understanding of so conceived concepts would consist in and what we can hope to gain from it. This is why we should probably stick to intensional considerations of language if we are going to explore Gödel's ideas about the theory of concepts.

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