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GROWING INTO DEDUCTION[1]

ABSTRACT: Psychologists have experimentally studied deductive reasoning since the beginning of the 20th century. However, as we will argue, there has not been much improvement in the field until relatively recently, due to how the experiments were designed. We deem the design of the majority of conducted experiments inadequate for two reasons. The first one is that psychologists have, for the most part, ignored the development of mathematical logic and based their research on syllogistic inferences. The second reason is the influence of the view, which is dogmatically still prevalent in semantics and logic in general, that the categorical notions, such as the notion of truth, are more important than the hypothetical notions, such as the notion of deduction. The influence of this dogma has been twofold. In studies concerning logical connectives in adults and children, much more emphasis has been put on the semantical aspects of the connectives – the truth functions, than on the deductive inferences. And secondly, even in the studies that investigated deductive inferences by using formal systems, the dogma still influenced the choice of the formal system. Researchers, in general, preferred the axiomatic formal systems over the systems of natural deduction, even though the systems of the second kind are much more suitable for studying deduction.
KEY WORDS: deduction, psychology of reasoning, proof-theoretic semantics

Introduction

Mathematical logic is still a very young discipline. Although the idea of a precise symbolic language was present already in Leibniz, logic truly begins in the 19th century with the works of George Boole and Gottlob Frege. Boole saw (propositional) logic as

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an algebra, where the words *and* and *or* behave like operations similar to multiplication and addition. The breaking point for logic was however 1879, the year in which Frege's *Begriffschrift* was published. For the first time in history, we have a precise notion of logical form of a proposition and a notion of formal language, where all the logically interesting connectives, as well as the quantifiers (the formal representations of the words *all* and *some*) appear in the form still present in contemporary logic.

While Boole's work is more in the field of *semantics* - the part of logic that deals with the meaning and interpretation of logical language, Frege's discoveries are closer to *syntax* - the part of logic that is concerned with the grammar of logical language and formal systems. For him, as well as for Bertrand Russell, all logic should be contained inside a particular formal system. The semantics of that formal system, if not contained in it somehow, is not even considered to be a part of logic. It seems natural to be more focused on syntax if one is interested in studying deductive inferences, which are in logic described by the formal systems. However, it is interesting that in a lot of psychological studies on logical reasoning, semantics had a prominent role. For instance, there were attempts to explain reasoning about logical connectives in reference to truth functions, and not in reference to inference rules (most famously by Jean Piaget). We will see later on how these explanations ran into problems.

Logic evolved most in the 20th century, especially during the first half. This period is associated with the great names of Gödel, Hilbert, Gentzen, Church, Turing, and others, to whom we owe the most important results. (We shall name here only a few that are most relevant for our subject). David Hilbert, together with his students, Paul Bernays and Wilhelm Ackermann, worked on the foundations of mathematics and gave an axiomatic formalization of classical and intuitionistic propositional as well as predicate logic, which later became a standard one. In the 1930's Gerhard Gentzen proposed another kind of formalization – *natural deduction*, which he proved to be equivalent to Hilbert's. Unlike Hilbert-style axiomatic systems, which contain many axioms and very few inference rules, natural deduction formal systems have many inference rules and very few axioms, if any. This enables hypothetical deductions – inferences from hypotheses, to be prominent in these formal systems. This is not the case in Hilbert-style systems, which deal primarily with the reasoning from axioms.

Hilbert is very famous for his program in the foundations of mathematics. He believed the whole of mathematics can be axiomatized with a set of axioms that could be proven to be consistent. However, Kurt Gödel showed this cannot be done by his famous incompleteness theorems. The theorems state that a consistent formal system based on a recursive set of axioms that contains enough arithmetic cannot be complete. In particular, it cannot prove its own consistency. Although Gödel is famous for the incompleteness theorems, maybe even more important for the development of logic is his proof of completeness of first-order predicate calculus. Gödel believed that the question of completeness is in the heart of logic. Some formal system is complete if there exists a precisely describable correspondence between this formal system and a mathematical model. For a logician, in particular, it is important to prove that the list of theorems provable in a formal system is complete with respect to the logical truths. A completeness proof is a kind of assurance that computations done inside the formal system are not arbitrary manipulations with symbols, but really capture a structure with solid mathematical ground. What's more, they show that nothing relevant about the structure has been left out by the system. The results of this kind are highly significant because they justify a new, syntactical way of dealing with the structure in question.

In logic textbooks one comes across claims that Aristotle's work has set the foundations of logic: „Aristotle's logic, especially his theory of the syllogism, has had an unparalleled influence on the history of Western thought" (Smith, 2019). In Aristotle's famous work, *Organon*, there is a list of deductive inferences called syllogisms that, from the viewpoint of modern logic, constitute a small fragment of first-order predicate logic. There is no argument that this list is exhausting or that it is enough for proving all the logical truths.[2] In Aristotle, the idea of completeness is nowhere to be found - it is not even considered to be a problem. Aristotle's merit is that he recognized that deduction is not related to the content, but rather to the form of a proposition that is determined by the words *all* and *some*. However, in his work, the notion of logical form is not accurate or general enough. Aristotle's analysis presupposes that every proposition is of the 'subject-predicate form', so he does not take into consideration propositions built by the logical connectives. From the standpoint of modern logic, which recognizes the fundamental role of connectives in deductions and formulations of logical truths, this is a considerable deficiency. It is also one of the reasons why syllogistic logic is not suitable for dealing with deductive inferences. Another reason is that it is not formal. Even though Aristotle did realize that propositional form is what is fundamental for deduction, he did not find the tools to represent this form precisely. Given that formality and accuracy that lack in Aristotle's work are what characterizes logic today, it is hard to say that logic truly begins with Aristotle.

It is less known that there are other thinkers in antiquity whose discoveries were more significant for the development of logic as a mathematical discipline than maybe all of Aristotle's. Philo of Megara from the Stoic school discovered the material implication, which provided a very successful analysis of the conditional propositions and was, among other things, responsible for the great success of modern logic. However, the value of his discovery was not appreciated until many years later, because in the Middle Ages logic did not progress much from Aristotle's initial ideas.

We believe that the case of logic should not be considered as much different from the case of physics. Although the idea of both disciplines stems from Aristotle, they

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2 For a different view see (Corcoran, 1972)

both became what they are today only after being mathematized. In case of logic, that happened not with *Organon*, but with the works of Boole and Frege. This fact has not been trully recognized in the psychology of deductive reasoning. There, the syllogistic approach has been prevalent for a very long time and is unfortunately still persistent. In what follows we shall present a brief sketch on the history of psychological experiments on deduction arguing that the syllogistic approach has hindered the research and oriented studies in the wrong way.

Experiments with syllogisms

Nearly all the early psychological research on deduction, which began in 1908 with Gustav Störring (Störring, 1908), concerned itself with syllogisms. Syllogisms are inferences with exactly two premises and one conclusion. Their premises and conclusion can only be sentences of some of the following forms: *All A are B*, *Some A are B*, *No A are B,* and *Some A are not B*. The conclusion of a syllogism states a relation between two terms[3] that is not explicitly stated in the premises. The subject of the conclusion must appear as subject or predicate of the second premise, and the predicate of the conclusion must appear as subject or predicate of the first premise. Premises also share one additional term that does not appear in the conclusion. A syllogism is correct if its conclusion is true whenever both its premises are true. An example of a correct syllogism looks like this:
All Bs are Cs.
Some As are Bs.
Therefore, some As are Cs.

Syllogistic logic describes and classifies patterns of syllogisms that differ by the form of their premises and conclusion and by the function that the subject and the predicate of their conclusion have in the premises. It also determines which are the patterns of valid inferences and specifies some principles that make them valid.

The experiments that use syllogisms test the ability of examinees to distinguish between valid and invalid syllogistic inferences. The subjects are presented with the examples of inferences and they are asked to assess their logical validity. Alternatively, they are given just the premises and asked to choose a conclusion with which these premises form a valid syllogism. What is measured is the percentage of correct answers and sometimes also the response time.

The most noticeable fact about the results of the experiments with syllogisms was that subjects' performance varied greatly from problem to problem. For example, most of the subjects incorrectly assessed as valid the syllogisms of the form *All As are Bs,*

3 In this tradition 'a term' is used as a general name for the subject or the predicate of a proposition

Some Cs are not As; Therefore, some Cs are not Bs. On the other hand, the valid syllogisms of the form *All As are Bs, Some Cs are As; Therefore, some Cs are Bs*, were in most cases evaluated correctly. A hypothesis that could explain these results is that deductions represented by different patterns of syllogism are of various levels of complexity. Some are rather basic, and they are evaluated with more success, while others are complex and more difficult to evaluate. Such an explanation might initiate the analysis of the patterns of syllogisms and their comparison that would further test this hypothesis. Psychologists did not, however, draw such a conclusion, possibly because they did not find in syllogistic logic the tools to analyze deductions and determine the relations between them.

The percentage of correct answers in the experiments was generally low. A lot of times subjects made mistakes and their reasoning was logically incorrect. This was the reason why the research on syllogistic reasoning from the 1920s to the 1950s was mostly an attempt at explaining the errors in reasoning. Given a large number of mistakes and inconsistencies in the subjects' assessment of syllogisms, it became widely accepted that in assessing the validity of a syllogism subjects do not reason by the rules of logic. So, psychologists made various hypotheses trying to explain the reasoning that is behind the answers subjects gave. For example, one of the hypotheses was that the presence of a particular or a negative statement in the premises makes subjects believe that a particular or a negative conclusion is correct (Woodworth & Sells, 1935). Another explanation was that subjects base their judgment on the content of the propositions in a syllogism and on their prior belief that its conclusion holds (Janis & Frick, 1943; Lefford, 1946). The researchers also believed that the errors lie in the interpretation of the premises. The subjects showed tendency to ascribe meaning to the premises not implied by their content, but by the Gricean language implicatures (Ceraso & Provitera, 1971; Wilkins, 1928; Woodworth & Sells, 1935), and to take the premises to imply their converses (Ceraso & Provitera, 1971; Wilkins, 1928; Chapman & Chapman, 1959).

After the emergence of artificial intelligence and information-processing psychology in the 1960s, syllogistic reasoning was studied using models. This gave rise to the so-called *mental-model theories* of syllogistic reasoning that were supposed to account for the correct as well as for the incorrect answers the subjects gave. According to these theories, people reason about syllogisms by constructing models of the states of affairs described in the premises, combining them and judging if the result is also a model of the state of affairs described in the conclusion. Models can be understood as Venn diagrams (cf. Erickson, 1974; 1978) or in some other way (cf. Johnson-Laird, 2001). The main problem with the models specialized for syllogisms is that they cannot be applied outside syllogistic reasoning.

Besides mental-model theories, there are also *heuristic* and *logic-based theories* of syllogistic reasoning. Heuristic theories explain how people evaluate syllogisms using some general principles of reasoning. According to one of these theories, they use principles concerning probabilities by which they evaluate the conditional probability of the conclusion (the probability that something is B if it is A) given the probabilities of the premises (Chater & Oaksford, 2001).

Logic-based theories, on the other hand, describe the reasoning about syllogisms as applications of inference rules or axioms related to the universal and existential quantifier (such as, for instance, natural deduction elimination and introduction rules). However, we have mentioned that such theories have in general been neglected because it turned out that reasoning about syllogisms was most of the time logically incorrect.

Having in mind various hypotheses and theories that psychologists have made concerning the reasoning with syllogisms, it seems that the focus of their experiments was not finding out if people reason deductively and according to which rules, but rather explaining the kind of reasoning - deductive or not, which the subjects used to solve the tasks and puzzles given in the form of syllogisms. Moreover, they were mostly interested in explaining the errors subjects made, and consequently, the aim of their experiments drifted away from studying deduction.

We believe that one should distinguish between research on deductive reasoning on the one hand and research on reasoning in general on the other. While reasoning can be correct or incorrect, deductive reasoning can only be deductive if it is correct – there is no such thing as incorrect deductive reasoning. It makes sense for non-deductive reasoning to be based on probabilities, expectations, prejudices, etc. Deduction, however, must be based on logic. If a deduction is not in general describable in a logical format (by some set of axioms and inference rules) it is not a deduction at all. Therefore, it should be precisely distinguished between the two very different questions, one being „what are the rules that govern deductive reasoning" and the other one being „what are the rules that make people not reason deductively (but by some other means)".

In the psychology of deduction, the distinction has not been explicitly defined and we believe it to be an important one. We also believe that there is no special reason for syllogistic inferences to play a prominent role in psychological research, especially in the research concerning the first question. What can be inferred from the results of the aforementioned experiments is that people in many cases do not deduce when they reason about syllogisms. Therefore, syllogisms are not of much use in studying deduction. They do not cover all possible cases of deductive inferences and, besides that, the inferences they do cover do not seem to be fundamental in a logical sense. By demanding that the propositions that make up a syllogism be of a specific form, syllogistic logic greatly diminishes the number and diversity of inferences that are its subject. The conviction that syllogistic logic can account for all deductive inferences is probably a consequence of the traditional view that all propositions must be of the 'subject-predicate form'. This belief has changed during the last century or two. Since Frege, who characterized a sentence as the most important linguistic expression, it has been accepted that the meaning of words should be explained in terms of the meaning of sentences in which they appear and not the other way around. This view became known in the philosophy of language as *the context principle*. The change of perspective was very important for the development of logic because it opened the possibility of considering sentences that are not of the 'subject-predicate form' and studying inferences involving these sentences. In that manner, modern logic gave us the tools to deal with much wider range of inferences. Its propositional fragment deals with inferences containing propositional connectives, and its other fragment - first-order predicate calculus, with inferences containing the quantifiers *all* and *some* in addition. Syllogistic inferences fall under those of the second kind. „The traditional theory, in fact, is that fragment of the predicate calculus in which four forms of proposition are selected for special study, it being assumed also that the terms appearing in these forms are not empty" (Lemmon, 1965, p. 177). With that assumption, all 24 valid patterns of syllogism are derivable in predicate logic, where they are dealt with in a precise and systematic way. From the perspective of modern logic, Aristotle's theory of syllogisms is not only surpassed but also highly redundant.

However, psychological research on deductive reasoning continued to focus on syllogisms (some more recent studies are Johnson-Laird & Steedman, 1978; Evans, Barston & Pollard, 1983; Evans, Handley, Harper & Johnson-Laird, 1999). Psychological studies that investigate deduction in kindergarten children also use mostly syllogistic logic as a model and very rarely deal with connectives (see for example Hawkins, Pea, Glick & Scribner, 1984; Dias & Harris, 1988; Johnson-Laird, 1980; Woodworth & Sells, 1935). Given that modern logic, which provides a much more efficient and precise method for the analysis of deductive inferences, has been developed more than a century ago and has proved its worth by its results and influences, we have to conclude that „the concentration of interest on the syllogism … is the symptomatic of the backward state of knowledge in this area" (Johnson-Laird, 1975, p. 37).

Logical connectives as truth functions

In the 1960s and the 1970s, psychological research on deduction became more up to date with modern logic and psychologists started to accept the insights that the development of logic brought. Accordingly, psychological research began to focus on the connectives. However, the emphasis was more on the semantic aspects of connectives than on deductions in which they partake. This choice was probably influenced by the dominant view on the meaning of logical language known as *model-theoretic semantics*. This view takes propositions to be the names for truth values, and logical connectives to be the truth functions. Therefore, in most of the experiments, logical connectives were understood as truth functions and their role in deductions as based on this interpretation.

Many experiments have been conducted with the goal of testing whether people understand and use logical connectives in accordance with their truth-functional interpretation (for example, Paris, 1973; Osherson & Markman, 1974; Braine & Rumain, 1981; Peel, 1967; Suppes & Feldman, 1971). In the majority of experiments, subjects were asked to determine the truth value of a proposition A*B, where * is a binary logical connective, based on the truth values of the propositions A and B. The subjects' performance was relatively poor. Even though they used the connectives with confidence, their understanding did not seem to completely agree with the truth-functional interpretation. The reason for this might be that people acquire the meaning of logical connectives not by learning their truth tables, but by understanding the inference rules in which they participate (cf. O'Brien, Dias, Roazzi & Braine, 1998). The fact that the percentage of mistakes was particularly high in the case of implication speaks in favor of this hypothesis. Namely, the difference between the truth-functional understanding of implication and the one based on its role in deduction is substantial. Implication is tied to deduction in a particular way: *if* signifies that a supposition is made and that consequences are deduced from that supposition. Hence, an implicational proposition can be taken to state that there is a deduction from its antecedent to its consequent, and moreover, this can be regarded as the essential part of the meaning of implication.

The view that logical connectives are defined by the role they have in deductions is known by the name of *proof-theoretic semantics* (see Schroeder-Heister, 2012a). This relatively new approach became an alternative to the model-theoretic view. However, the model-theoretic approach is in logic still the dominant one, and psychologists were so influenced by it that they tended to treat deductions in which logical connectives participate as dependent upon their truth-functional interpretation.

This view on deduction is apparent in the investigations of Jean Piaget, one of the most influential psychologists that studied cognitive development in children and adolescents. At the end of the 1950s, he became interested in the 'formal thought' that should be developed between the age of 11 and 15 and that is characterized by the use of truth functions, or 'propositional operations' in Piaget's terminology. According to Piaget, the use of truth functions by adolescents is manifested in their ability to assume hypotheses and to deduce their consequences. Piaget's studies on the development of formal thought were based on a series of physical and chemical experiments conducted by Inhelder (Piaget & Inhelder, 1958). What he recognized as hypothetico-deductive reasoning in these experiments was the subjects' ability to distinguish between all the possible combinations of factors in a particular experimental situation and to reason about their effects. He found that in the process of reasoning about hypothetical situations, subjects use all sixteen binary, and also some ternary, truth functions.

In the process of reasoning, subjects also pass from one truth function to another, and it seems that these transitions are what Piaget thought deduction consists of. What justifies them, in his view, are the relations between the truth functions. These relations form a particular algebraic structure - a *commutative group* or a *lattice* (Piaget, 1957, p. 33). Piaget believed that subjects who use truth functions intuitively construct an algebraic structure and accept its laws that enable them to reason deductively. So, he attempted, by the means of an algebra of truth functions, to construct a theory of deduction that would explain the experimental findings. This way of understanding deduction is very different from how logicians understand it. In logic, deduction is understood as something tied to logical connectives and justified by the properties they have as part of a formal system, and not by the properties of their interpretations. Piaget, however, does not make a clear distinction between the connectives as linguistic entities, and truth functions by which they are interpreted. However, there are natural language expressions only for some truth functions. Those are the connectives *and*, *or*, *not*, *if-then*, *if and only if*, that also have a significant role in deduction, which can be precisely characterized by the inference rules. Piaget takes them to be simply the names of truth functions and does not seem to be interested in the syntactic properties that, according to logicians, describe their deductive behavior. That is why he considers these connectives to be on a par with all the other expressions standing for some truth function. Even though he was aware that logicians understand deduction in a way much different from his own, Piaget seems to have thought that his account on deduction corresponds better with how people actually reason. The role that connectives have in reasoning is, in his opinion, determined by the algebraic properties of the functions they denote, and not by their syntactic properties described inside formal systems: „the logic of the real subject is isomorphic to an algebra as *calculus* more than to a formalized language" (as cited in Parsons, 1960, p. 81, from *Logique et equilibre*).

However, the truth-functional interpretation of the connectives cannot in fact account for their deductive behavior. For example, it does not tell us how an implicational proposition can be proved or what can be deduced from a disjunctive proposition. The role that logical connectives have in deduction can only be explained by the inference rules of a formal system. Therefore, taking the truth-condition approach for describing deduction and deductive properties of the connectives may not be the best choice. As it turns out, the connectives that are closely tied to deduction, such as intuitionistic implication, are not truth-functional (Došen, 1989).

We believe that the choice of the researches to focus on the semantics and not on the formal deductions in studying logical reasoning was influenced by the dominant view in logic according to which categorical notions have precedence over hypothetical ones (Schroeder-Heister, 2012b). The consequence of this dogma (which can also be formulated as the view that descriptive language uses should have precedence over prescriptive ones)[4] is that asserting, as a categorical notion, is more important than deducing, as a hypothetical notion, and that the later should be accounted for in terms of the former. Deduction is thus usually defined as truth preservation.

This view leads to reducing deduction to the consequence relation and disregarding its structure. Taking deduction to be the consequence relation leaves us with no way of distinguishing between different deductions with the same premises and conclusion (see, for example: Došen, 2011). Drawing the distinction is, however, important for the psychology of reasoning. Different deductions can be of different levels complexity. In testing deductive reasoning, it is preferable to use basic deductions that are not reducable to simpler ones, and avoid those that are too complicated and that depend on additional skills, such as good memory or concentration. To determine which deductions are basic, logicians analyze their structure inside formal systems. In the next chapter, we will argue that the choice of the formal system is a focal point in testing informal inferences, and that the results of the experiments that test them are highly dependent upon it.

Informal and formal deductions

Not all deductive tasks are the same. While some are difficult and demand more skill, some are rather easy and simple. It is not always the case that deductions people find intuitive and easy are going to be basic or easily derivable inside a particular formal system. Take, for instance, the formal proof of p ® p inside Hilbert's system for classical propositional logic. It is somewhat complicated (compared to the difficulty of the intuitive task). First, one needs to instantiate the axiom schema (A ® (B ® C)) ® ((A ® B) ® (A ® C)) in the following manner: (p ® ((p ® p) ® p)) ® ((p ® (p ® p)) ® (p ® p)). From that formula, together with p ® ((p ® p) ® p), which is an instance of the axiom schema A ® (B ® A), it follows by modus ponens, that (p ® (p ® p)) ® (p ® p). Since, (p ® (p ® p)) is also an instance of A ® (B ® A), it follows again by modus ponens, that p ® p.

It is rather obvious that people do not reason this way if asked to justify the conclusion that p ® p. But why is that so? Do they just happen to reason illogically?

In the 1960s and 1970s, psychologists began to study deduction by comparing the informal inferences of people not trained in logic to proofs inside a formal system for propositional logic. Many psychologists tried to dispute the view that formal logic bears a relation to human reasoning based on the results of these experiments. The researches were unable to find a formal calculus that describes subjects' reasoning (see Wason & Johnson-Laird, 1972, p. 245), and they tried to explain this by claiming that formal systems and informal deductive reasoning have different purposes:

4 For a more detailed account see (Maksimović, 2016)

A further divergence between logical calculi and the inferential machinery of everyday life concerns their respective functions. Calculi are devised primarily for deriving logical truths. The aim of informal inference, however, [...] is to pass from one contingent statement to another. (Johnson-Laird, 1975, p. 16)

In other words, the researchers in (Wason & Johnson-Laird, 1972), noticed that the subjects were prone to reason from hypotheses by using many different rules of inference, while it was not natural for them to reason from axioms. The reason they were skeptical about the possibility of representing informal deduction inside a formal system, is that they were dealing with axiomatic formal systems exclusively.

Axiomatic, Hilbert style formal systems have many axioms and very few rules of inference. Since, by default, axioms are given the role of premises of an inference, it is not immediate or very natural to represent *hypothetical* deductions – that is, deductions from hypotheses, inside those systems. However, although in axiomatic formal systems one infers from *logical truths*, by default, the aim of a formal system, in general, has nothing to do with truth; its purpose is the same as the purpose of the informal deductive apparatus - to deduce. It is just that inside a formal system deductions are made more rigorous to avoid mistakes and this is sometimes done at the cost of divergence from intuitive reasoning. To what extent the formal deductions diverge from the informal ones is highly relative to the choice of a formal system. In the case of Hilbert style formal systems, the divergence is significant. This makes them hard to work with in practice. That is why when working inside such a system, a logician or a mathematician will first prove the *deduction theorem* and then a list of theorems that enable deductions inside the system to appear more natural and intuitive.

Not every formal system is axiomatic, however. In (Gentzen, 1934) and in (Jaskowski, 1934), propositional as well as predicate logic is formalized inside natural deduction calculus. Natural deduction systems can have different formats but they all have many rules of inference and very few axioms (if any). In (Gentzen, 1934), for instance, the formal system for classical propositional logic contains a set of pairs of introduction and elimination rules for every connective, with an additional classical assumption $A \vee \neg A$. For example, the rule for introducing conjunction states that from the premises A and B, the conclusion $A \wedge B$ can be inferred, while the elimination rule states that any of the two conjuncts A and B can be inferred from $A \wedge B$. According to the implication introduction rule, if B can be deduced from A, then $A \rightarrow B$ holds, independently of whether A holds or not (in logical terms one also says that this hypothesis is *discharched*). If both $A \rightarrow B$ and A hold, the conclusion B can be inferred by the rule for implication elimination.

Other natural deduction systems contain different rules, such as the disjunctive syllogism, or have more complex forms of introduction and elimination rules.

Having a lot of inference rules and few axioms makes reasoning from hypotheses prominent in natural deduction; hypothetical deductions are given priority over categorical ones. That is one of the reasons why natural deduction has, unlike Hilbert type systems, proved rather well in practice – people find it a lot easier to learn and apply.

The idea that natural deduction describes the way people informally deduce stems from Gerhard Gentzen. Gentzen called the system he formalized *natural* deduction because he believed it represents the way mathematicians reason when proving theorems. Consider, for instance, the proof of p → p inside Getzen's natural deduction system. Suppose that p. Since, from p, one can infer that p, the implication p → p follows by the implication introduction rule, and the hypothesis p is no longer assumed. This proof may have slightly more structure than the intuitive one, but it seems close enough.

However, in the literature on the psychology of reasoning, there was no mention of natural deduction before the middle of the 1970s. But why is that so? We believe that the main reason is the dogma of the precedence of categorical over hypothetical, which has been the dominant view in logic and has had a huge influence on the psychology of reasoning. A consequence of this dogma is taking the notion of categorical proof – a proof without hypotheses, as more important than the notion of hypothetical proof – a proof with hypotheses. (In general proof theory, it is actually the other way around. Hypothetical proofs are given primacy because categorical proofs can be seen as their special cases – as proofs from the empty set of hypotheses.) This conviction has influenced the design of the experiments by giving precedence to the axiomatic systems as opposed to the systems of natural deduction, which in turn led to poor and inconclusive results.

In (Johnson-Laird, 1975) we find a proposal to test if natural deduction is the right format for describing deduction as a psychological process, because of its rule-based hypothetical approach. In the 1990s psychologist Lance Rips conducted a series of experiments to test this conjecture (Rips, 1994).[5] His examinees were mainly students and people not trained in formal logic. The results of Rips' experiments show that when people reason informally, their deductions are very similar to the corresponding hypothetical proofs in a system of natural deduction.

Rips uses a computer simulation of a person that deduces. Simulation is programmed according to an automated prover that is based on a natural deduction system for propositional logic. This system is similar to Gentzen's but with some additional rules such as the disjunctive modus ponens or the disjunctive syllogism. The rules used by the simulation are of two kinds: forward and backward rules. Forward rules, such as modus ponens, go from the premises to the conclusion. Backward rules go the other way around, in a sense (see table 4.1. in Rips, 1994). The elimination rules are classified as forward rules and the introduction rules as backward rules. The forward rules are considered easier to apply because in applying backward rules one needs to plan out the derivation, which requires more memory and skill.

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5 Other studies similar to (Rips 1994) are: (O'Brien, 1987); (Braine, Reiser & Rumain, 1998); (Braine & O'Brien, 1998)

In the experiments, subjects are asked to evaluate the logical validity of an inference, or they are asked to remember the inference steps or to decide whether a certain inference rule can be derived from a given set of rules. Similar tasks are given to the deduction model (the automated theorem prover) which works in the framework of natural deduction. What is estimated is the level of difficulty of the task according to the model and according to the subjects. What Rips has found is that these levels coincide. There is a certain correspondence between informal deductions on the one hand and hypothetical proofs in the system of natural deduction on the other.

Rips used the obtained results to conclude that deducing from hypotheses and using inference rules rather than axioms is a characteristic of deduction as a psychological process and is as such inherent to the way people informally deduce. These findings explain why the subjects' performance was so poor on the experiments that used axiomatic formal systems.

The findings of Rips' research and its comparison with the research based on axiomatic formal systems suggest that the results are very sensitive to the formal format which one uses to explain informal reasoning. It appears that not all formal systems are on a par, some (such as natural deduction) are more in accordance with informal deductive reasoning than others.

Even after Rips' experiments the thesis that informal logical reasoning can be described by some set of inference rules has been challenged by the *mental model school*, on the basis of the fact that people often make mistakes when trying to deduce, and do not always recognize some deductions as valid. However, the fact that people are prone to making mistakes when solving complex deductive tasks does not speak against the view that when people in fact deduce (correctly) they deduce according to these rules. It has been observed that people are not so much prone to making mistakes about the simple natural deduction rules as they are to making mistakes about some more complex deductive tasks. And on the other hand, what if people sometimes do make mistakes concerning introduction and elimination rules? It need not be true that all people have a clear and complete understanding of the connectives. It is a fact that not all people are good at deducing. In claiming that natural deduction describes informal deductive inferences, Gentzen's idea was just that when people deduce (in mathematics) they deduce according to the natural deduction rules and not that people always reason deductively and never make mistakes.

Deductive reasoning in young children and some philosophical considerations

For the psychology of reasoning, it is also interesting to inquire in which period of the cognitive development deductive reasoning appears, and if there are some deductive tasks that even very young children are able to perform. We believe that the choice of the formal format is even more important for testing deduction in young children than in adults. Deductions being tested must be very simple because the memory capacities of young children are not yet developed in full. As we have already pointed out, natural deduction systems have the advantage over axiomatic Hilbert style formal systems in containing deductions that are more natural and more easily mastered. The most basic deductions inside them consist of only one application of some inference rule. Compared to the other natural deduction formats, Gentzen's natural deduction inference rules are the simplest, which is why we think the experiments with children should be based on them.

Using natural deduction as a tool to investigate logical capabilities in children led to some very interesting findings. The results of (Braine & Rumain, 1981), (O'Brien, Dias, Roazzi & Braine, 1998), and (O'Brien & Shapiro, 1968) suggest that very young children (5 - 8 years of age) are in fact capable of making some deductive inferences, even though they have not yet reached the phase of 'formal operations' which is in Piaget's theory the period in adolescence when logical capabilities are developed. Among the natural deduction inference rules that were tested in these studies, the ones that had the format as in (Gentzen, 1934) were disjunction elimination (tested on the children aged 5 to 8) (Braine & Rumain, 1981), and modus ponens (tested on the children aged 6 to 8) (O'Brien & Shapiro, 1968). Remarkably, most of the time children answered correctly (around 73% of correct answers for disjunction elimination and around 75-81% of correct answers for modus ponens), although their performance was rather poor on the tests involving truth conditions.

These results turned out to be quite interesting, not only for child psychology but also for semantical considerations. The standard semantics of classical logic is model-theoretic. Meanings are seen as set-theoretic entities and parts of logical language as just labels for these entities. In this tradition, logical connectives are understood as truth functions. An alternative view on meaning of logical connectives is proposed by proof-theoretic semantics. According to this view, a connective is defined by the appropriate introduction and elimination rules. It is interesting to inquire whether the meaning of natural language logical connectives can be explained similarly. A way to approach this question would be to test how children understand the connectives at the time when they just start to use them. If it turned out that at the same time children learn to deduce according to the natural deduction rules, this would lend support to the thesis of proof-theoretic semantics.

Psychological studies show that children start to make first complex sentences at an early age of 2 to 3. The study presented in (Clancy, Jacobsen & Silva, 1976) has shown that conjunction is the first connective children learn to use in each of the four languages they have investigated. They start to use disjunction soon afterwards, and at the age of 2 years and 8 months children spontaneously utter their first conditional sentences (see: Bowerman, 1986; Reilly, 1986; Bloom, Lahey, Hood, Fiess & Lifter, 1980; Peel, 1967; Paris, 1973; Chierchia, Crain, Guasti, Gualmini & Meroni, 2001; Scholnick & Wing, 1991).

A recent research (Kostić, Maksimović et al., 2018) provides evidence that very small kindergarten children (3 to 5,5 year-olds) are in fact capable of understanding and using Gentzen's introduction and elimination rules for the connectives *and*, *or* and *if-then*. While they are most successful in introducing and eliminating conjunction (around 90% of correct answers), they have some difficulties with introducing implication (only around 30% of correct answers). But also in the case of implication it seems that children found it more troublesome to understand what is their task or to remember the premises than to actually deduce the conclusion. Nevertheless, the fact that a significant number of children gave correct answers and the fact that they have used the rules spontaneously to solve the tasks in the experiment is quite remarkable. It lends support to the thesis that the introduction and elimination rules are closely tied to the meaning of the connectives since children start to use both at around the same age. Similar conclusion has been drawn for the case of implication by (O'Brien, Dias, Roazzi & Braine, 1998).

It is also interesting that the children were generally better with the elimination rules than with the introduction rules. An explanation for this can be found in the fact that before learning to come up with complex sentences on their own, children hear the adults pronounce them (for instance: „If you eat your soup you will have the candy"), and learn how to eliminate the connectives in order to draw conclusions from these sentences ("I will have the candy"). The discrepancy is somewhat proportionate to linguistic competences. It is most prominent in very young children (3-year-olds) and becomes less prominent with age.

The results of (Kostić, Maksimović et al., 2018) and similar studies change how one perceives the role of deduction in the acquirement and development of language. Since Frege and early (Ludwig) Wittgenstein, asserting has been perceived as the central language function and a proposition as the principal unit of meaning. The relation between deduction and meaning of linguistic entities has not been considered as particularly significant; it has not been seen as immediate at least. For Frege, there are two aspects of meaning – *sense* and *reference*. While both are purposeful in natural language, only reference is relevant in logic. The philosophical position inspired by model-theoretic semantics takes this view even further – it disregards completely the sense of linguistic expressions. The paradigmatic relation between language and its meaning is taken to be reference. Meanings are seen as entities and linguistic expressions as labels for these entities.

In *Philosophical investigations* Wittgenstein criticizes this position. His *Investigations* start with a quote from St. Augustine describing the process of language acquisition from his personal experience: „When grown-ups named some object and at the same time turned towards it, I perceived this, and I grasped that the thing was signified by the sound they uttered since they meant to point *it* out". (Augustine, *Confessions I,* ch. 8)

Wittgenstein comments on this quote in the following manner:

"These words, it seems to me, give us a particular picture of the essence of human language. It is this: the words in language name objects – sentences are combinations of such names. – In this picture of language, we find the roots of the following idea: Every word has a meaning. This meaning is correlated with the word. It is the object for which the word stands." (Wittgenstein, 1958, p. 2)

In the rest of the book, Wittgenstein criticizes this position and changes the perspective entirely with his famous dictum *Don't ask for the meaning, ask for the use*. For Wittgenstein, what constitutes the meaning of a word is how it is used, not the entities it signifies. However, the use of words is something prescribed by the rules (of language) related to the activity in which these words are used (Maksimović 2016, p. 27). If the main activity regarding logical connectives is deducing, then indeed, the rules that define their meaning should be the rules of inference such as the introduction and the elimination rules of a natural deduction calculus.

While traditional model-theoretic semantics goes along with Frege and early Wittgenstein in emphasizing descriptive language uses such as asserting and naming, the proof-theoretic semantics is following the later Wittgenstein's views. More emphasis is put on prescriptive language uses – the rules of inference, which are seen as constitutive for the meaning of logical connectives. From this perspective, deduction is not considered to be an activity reserved for mathematicians and scientists, but an activity that constitutes the use of some very important words of natural language – logical connectives. Although the model-theoretic view on meaning is still the default one in logic, some of the experimental results we have mentioned here speak in favor of the proof-theoretic approach.

Summary and conclusions

The goal of this paper was to give an overview of the experimental investigations on deduction and their results. In the majority of the experiments, the subjects performed poorly. They did not seem to reason in a way that was expected (according to the truth-functional understanding of connectives or the particular formal deductions), and this led to inconclusive results. We have tried to show that this was not because people in general reason illogically, but because of the way how the experiments were designed.

The development of modern logic has brought a great improvement in the understanding of deduction and it seems reasonable that the experiments on deductive inference should comply with it. However, psychological research, in general, was not very moved by the insights of modern logic. Most of the experiments that tested deductive reasoning focused on syllogistic inferences, the study of which belongs to, so to say, the prehistory of logic. Also, the researches that did accept the modern approach have been mostly influenced by the semantics of logical connectives. From the standpoint of logic, however, what is most relevant for deduction is the syntax of logical connectives described by formal systems. But, even in the psychological research based on formal systems, there is still the danger of choosing a formal system that is not particularly adequate for representing deductions. Psychologists that based their research on axiomatic formal systems found out, not surprisingly, that they do not describe the way people actually deduce. But there are also formal systems of another kind – natural deduction formal systems, that are more suitable for studying deduction, and that are incomparably more successful at capturing informal inferences. However, they did not receive much attention from psychologists until relatively recently. We think the reason for this could be the fact that these systems give primacy to hypothetical over categorical deductions. This is against the dogma, present also among logicians, that categorical notions, such as the notion of truth and proof, have primacy over hypothetical ones, such as the notion of deduction. We have tried to show that going against the dogma by choosing natural deduction over other formal formats for the experimental tests, has led to discoveries significant, not only for psychology, but for philosophy and semantics as well. These discoveries suggest that deduction has an important role in understanding logical connectives in natural language. We believe that this could be an inspiration to philosophers and psychologists to further investigate the boundaries between deduction, meaning, and cognitive development.

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Ka dedukciji
(*Apstrakt*)

Psiholozi su počeli da se eksperimentalno bave deduktivnim zaključivanjem početkom 20. veka. Ipak, zbog načina na koji su eksperimenti bili osmišljeni, nije bilo značajnih pomaka u toj oblasti sve do relativno skoro. Smatramo da postoje dva glavna razloga zbog kojih dedukcija često nije bila ispitivana na adekvatan način. Prvi je taj što su psiholozi u velikoj meri ignorisali razvoj matematičke logike i bazirali svoja istraživanja na silogizmima. Drugi razlog je uticaj gledišta, koje i dalje preovladava u semantici i logici uopšte, da su kategorički pojmovi, kao što je pojam istine, važniji od hipotetičkih pojmova, kao što je pojam dedukcije. Uticaj te dogme na psihološka istraživanja je bio dvostruk. U studijama koje su se bavile shvatanjem logičkih veznika kod odraslih i kod dece, mnogo više značaja je pridavano semantičkim aspektima veznika – istinosnim funkcijama, dok su dedukcije stavljane u drugi plan. Sa druge strane, dogma je uticala čak i na istraživanja koja su pomoću formalnih sistema ispitivala deduktivno zaključivanje na taj način što je uslovljavala izbor sistema. Istraživači su uglavnom preferirali aksiomatske formalne sisteme naspram sistema prirodne dedukcije, iako su se za izučavanje dedukcije potonji pokazali kao daleko adekvatniji.

KLJUČNE REČI: dedukcija, psihologija zaključivanja, dokazno-teorijska semantika